

What Makes a Whip Crack?

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What makes a whip crack is the tip exceeding the speed of sound. The crack is a miniature "sonic boom" just like the one an airplane creates when it exceeds the speed of sound. The speed of sound at sea level is approximately 760 miles per hour or about 340 meters per second (it varies a few miles per hour depending on temperature and air pressure) Some people have trouble believing that physics would allow a person - even a very strong person - to throw anything at 760 miles per hour. Since it's easily demonstrable that even a child can crack a whip, these people believe something else must be making the sound. Perhaps the whip is hitting itself. In fact, rigorous physics shows that a sonic boom is exactly what happens. I will use three equations, some simple algebra and the concept of ratios to show this.

Kinetic Energy

Kinetic energy is the energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. Kinetic energy is an expression of the fact that a moving object can do work on anything it hits; it quantifies the amount of work the object could do as a result of its motion. The kinetic energy of a point mass m is given by:

$$\text{equation 1} \quad E = \frac{1}{2} m(V^2)$$

or the equivalent equation

$$\text{equation 4b} \quad V = \frac{\sqrt{2E}}{\sqrt{m}}$$

Where:

E = energy in Joules

m = mass in grams

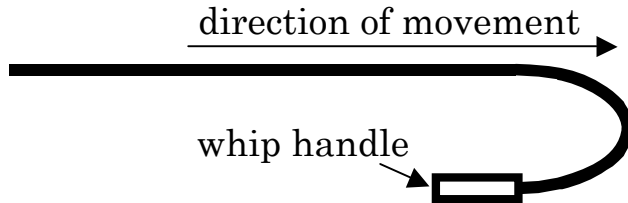
V = velocity in meters per second

We all know that $1/2$ is more than $1/100$, and that is the key to understanding what happens when a whip cracks. When a whip is thrown, the part that's moving, the part that 'holds' the kinetic energy shrinks, but the energy remains about the same. This is represented in equation 4b by the m in the bottom of the equation. As m gets smaller, the number represented by the equation, V , gets bigger. In the real world, this manifests as a dramatic increase in the speed of the tip as the whip approaches full extension. This is very scientific, and perhaps intimidating, but I did say I was going to use physics. I promise to make it as easy as I can. I have put the algebraic translation of the above equation at the end so those of you whose eyes glaze at the sight of such things will not be overly bothered.

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What really happens:

In a simple overhand whip throw, the whip starts with most of its length - it's mass - behind the person throwing it. The whip thrower brings his hand forward, imparting some velocity to the whip - let's say 25 miles per hour, or about 11.5 meters per second - and then his hand stops when his arm reaches full extension. This leaves a situation where the whip handle is stationary and most of the whip is moving forward, as shown in drawing 1.



drawing 1

The whip handle is being held by the person throwing the whip, and is more-or-less stationary. The part of the whip body represented by the upper section of the line is moving forward

Given that professional baseball players routinely throw baseballs at over 90 miles per hour, 25 miles per hour is a reasonable assumption for how fast a whip can be thrown, even by a couch potato.

For this discussion, let us assume the part of the whip that is moving has a mass of 6.2oz, or about 175 grams.

If we plug these numbers into equation 1, we arrive at:

$$\text{equation 1} \quad E = \frac{1}{2} m(V^2)$$

$$E = \frac{1}{2} 175(11.5^2) = 87.5 \times 132 = 11,550 \text{ Joules}$$

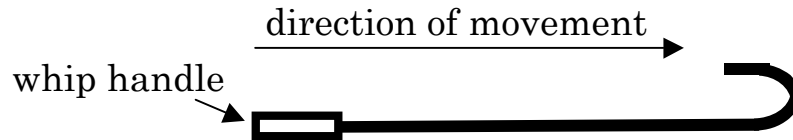
This is the kinetic energy in the moving part of the whip.

In the real world, some energy will be lost to internal friction as the bend moves down the whip and to resistance of the air to the moving whip. For this argument let's assume 15% of the energy is lost and the other 85% of the energy is transferred to, and concentrated in, the moving part of the whip. Taking our energy number from above, we have:

$$11,550 \times 85\% = 9,815 \text{ Joules}$$

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As the upper part of the whip and the curve in the body of the whip move, more and more of the whip is stationary because it is stretched out from the stationary handle, and less and less is in motion, as shown in drawing 2.



drawing 2

Much less of the whip body is now moving, and more is stationary.

For this discussion, let us assume the part of the whip that is moving in this drawing is just the cracker, and it has a mass of five one-thousandths (.005) of an ounce, or about 0.15 grams. In practice, as the bend in the whip approaches the tip, the mass of the moving part of the whip approaches zero, so we could pick any small number that makes our equations work, and simply wait a fraction of a second until that was the only part of the whip left moving. This also means that more of the length of lighter "thin" crackers will exceed the speed of sound as compared to heavier "fatter" crackers.

If we plug these numbers into equation 4b, we arrive at:

$$\text{equation 4b} \quad V = \frac{\sqrt{2E}}{\sqrt{m}}$$

$$V = \frac{\sqrt{2 \times 9,815}}{\sqrt{.15}} = \frac{140}{.387} = 362 \text{ meters/sec}$$

362 meters per second works out to about 810 miles per hour - well above the 760 mph required to create a sonic boom.

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Solving for Velocity (optional reading)

For this discussion, we need to solve for velocity as well as energy, so we need to translate the energy equation. If your eyes glaze over at the sight of algebraic translations, just assume I did it right and accept the discussion on pages 2 and 3. I only include my work in the name of rigor.

First multiply both sides of the equal sign by 2:

$$\text{equation 2a} \quad 2E = 2 \cdot \frac{1}{2} m (V^2)$$

The upper and lower 2s on the right cancel giving us:

$$\text{equation 2b} \quad 2E = m(V^2)$$

Then we divide both sides of the equal sign by m

$$\text{equation 3a} \quad \frac{2E}{m} = \frac{\cancel{m}(V^2)}{\cancel{m}}$$

the m 's on the right cancel giving us:

$$\text{equation 3b} \quad \frac{2E}{m} = V^2$$

Finally, we take the square root of both sides giving us:

$$\text{equation 4a} \quad \sqrt{\frac{2E}{m}} = V$$

or the equivalent equation:

$$\text{equation 4b} \quad V = \frac{\sqrt{2E}}{\sqrt{m}}$$

The key when looking at equation 4b is that the term m for mass is on the bottom of the fraction on the right side and the term v for velocity is on the top on the left

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Some More Optional Stuff

The astute reader will note that it's possible to combine equation 1 and equation 4b and our percentage of loss calculation into one equation in three variables:

$$\text{equation 5} \quad V_2 = \sqrt{\frac{m_1 \times 0.85}{m_2}} \times V_1$$

Where:

m_1 = the mass of the whip body in grams

m_2 = the mass of the whip cracker in grams

V_1 = the velocity the whip is thrown in meters/sec

V_2 = terminal velocity of the cracker in meters/sec

I leave it as an exercise for the reader to convert these numbers to English units if they desire to do so. The conversion factors are all readily available to anyone with a computer.

It can be amusing and enlightening to create a spreadsheet and play with the variables in this formula. It will rapidly become clear why longer and/or heavier whips are easier to crack, and why thin crackers with few strands are easier to crack than fat, fluffy crackers with many strands.

Bottom line is: long heavy whips with thin crackers will get more of their mass above the speed of sound and make louder cracks or crack with less effort.

What is perhaps not so clear from these calculations is how these variables effect accuracy and what happens when something other than 'perfect' throws are made.