Collapse of SRT 1: Derivation of Electrodynamic Equations from the Maxwell Field Equations

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Heaviside (1888, 1889) and Thomson (1889) first correctly calculated the scalar and vector potentials of a steadily moving point charge by transforming d'Alermbert's equation for the potential for a steadily moving charge into Poisson's form for a static charge by elongating a coordinate axis lying along the direction of the charge translation. They thus developed a way to solve dynamic problems like static problems, using an auxiliary equation in the form of Poisson's potential equation. The present authors use this ingenious mathematical approach to derive from Maxwell's field equations alone many useful electrodynamic equations, including auxiliary Lorentz transformation equations.

Key Words: Heaviside –Thomson Electrodynamics, auxiliary system, point charge

1. Introduction

Isaac Newton described his physics within the framework of absolute space and absolute time, independent of each other. According to Newton, even if the Universe were destroyed, absolute space and absolute time would still exist [1]. But Newton's views were very strongly contested by Leibnitz, who viewed space as the order of co-existent phenomena, and time as the order of successive phenomena. Both space and time would therefore be relative: if there were no phenomena, neither would exist [1]. Yet there exists the centrifugal force acting at the surface of our Earth. According to Newton, this indicates that the rotation of Earth is absolute. Hence, absolute space obviously exists. These opinions battled for centuries. Euler, Neumann, Maxwell [2] were influenced by the thoughts of Newton, but Berkeley [3], Boscovich, Stallo and Mach [4] shared the view of Leibnitz.

Mach argued that Earth rotates with respect to the fixed stars, and if a person does not see the fixed stars, he cannot settle whether Earth is rotating or not. Mach proposed that Newton should stop first the rotation of Earth. Then he should rotate the heaven of fixed stars around it. Now, in such a situation, if there would be no centrifugal force acting at the surface of Earth, one might conclude that the centrifugal force originates from the rotation of the Earth. As those feats could not be done, it cannot be said with certainty that the centrifugal force originates from the rotation of this planet. Mach believed that a new system of mechanics could also be developed to explain the centrifugal force acting on the surface of the Earth by the action of rotating stars by considering the Earth to be stationary. But he did not formulate such a system of mechanics.

After Mach, it was clear that the absolute space and absolute time of Newton are only his assumptions – the mental construction of Newton. But because Newton created a system of mechanics that can explain physical phenomena correctly, everyone in the scientific communities rightly believed in Newton's absolute space and absolute time. But from the end of 19th century, it was known that a good many experiments of electrodynamics were not in harmony with Newton's mechanics, *e.g.* **1)** when a charge moves, its mass increases with speed, which is not compatible with Newton's understanding of Nature; **2)** when a charge is accelerated in an electromagnetic field, Newton's sacred law $\mathbf{F} = m\mathbf{a}$ is violated, and the charge cannot exceed the speed of light under any circumstance; **3)** light propagates with speed c in free space, where Earth is moving with a very high velocity; therefore, the speed of light should change if measured on Earth, depending on the direction of Earth's movement. But with repeated experiments it was settled that the speed of light is the same c in all directions, if measured from Earth; **4)** from the consideration of Newton, the laws of reflection, refraction, diffraction and interference of terrestrial and astral light should differ. But no such difference was observed. **5)** As measured by the scientists, the speed of light propagating in a moving medium is $V = c/n + u(1-1/n^2)$, where n is the refractive index of the medium, and u is the speed of the medium. This is not in conformity with Newton's understanding of velocity.

To explain all these perplexing electromagnetic results on an *ad hoc* basis, H.A. Lorentz developed four real transformation equations that are not Newtonian in appearance. The transformation equations between two inertial frames S and S′ with translation at relative speed u along their x axes are: **i)** $x' = \gamma(x - ut)$, **ii)** $y' = y$, **iii)** $z' = z$, **iv)** $t' = \gamma(t - ux/c^2)$, where $\gamma = 1/\sqrt{1 - u^2/c^2}$.

It was Albert Einstein who deduced those same four equations from assumptions allowing space and time to be relative, and time to be a linear function of coordinates. His assumptions were: **i)** $x^2 + y^2 + z^2 = c^2t^2$, **ii)** $x'^2 + y'^2 + z'^2 = c^2t'^2$, commonly known as the principle of light-speed constancy in all inertial frames; **iii)** $x' = \gamma(x - ut)$, **iv)** $x = \gamma(x' + ut')$, commonly known as the principle of covariance of all physical laws; with **iv)** $y' = y$, **v)** $z' = z$, and with γ then derived from the stated principles. Thus within the framework of relative space and relative time, Einstein was able to create a new system of mechanics, which was the dream of philosophers and physicists for centuries. Therefore, Albert Einstein has been considered as one of the greatest philosophers and scientists of the 20th century.

But the first of Einstein's assumptions one is a derivation from Maxwell's wave equation, which is applicable only in free space, but not at all in other situations. The next three Einstein assumptions are absurd from any realistic viewpoint. Moreover, Einstein could not satisfy Mach's criterion; *e.g.*, when a radiating dipole moves on Earth, and an observer is at rest on Earth, there is transverse Doppler effect, which has been confirmed by experiments. But if the radiating dipole is at rest on Earth, and an observer moves in the same opposite motion, then from consideration of the Einsteinian idea of relative space, there should also be the same transverse Doppler's effect. But this could not be shown by any experiment.

Therefore, it is clear that, just as absolute space and absolute time were the mental creation of Newton, so were the relative space and relative time the mental creation of Einstein. The only difference is that Newton's assumptions are easy, independent, and plausible, whereas Einstein's assumptions are complicated, interrelated and absurd.

Following [5], we show below that all those electrodynamic phenomena, including later experiments like the Ives-Stilwell experiment (1938) and the experiment of Farley *et al* (1966), could easily and physically be explained from Oliver Heaviside's electrodynamics. This electrodynamics is a continuation of Newton's mechanics. In the last part of this paper, we have deduced, too, auxiliary Lorentz Transformation Equations from Heaviside's electrodynamics.

2. Electrodynamics of Heaviside and Thomson

Maxwell elegantly explained the nature of propagation of electromagnetic disturbances in free space. Oliver Heaviside, Thomson, and their contemporaries, proceeded to cast the potential problems of a system of moving charges into Poisson's form by the elongation of the coordinate axis in the direction of movement of the system of charges, taken to be the OX axis. Thus they developed a way of solving dynamic potential problem in a static format by using a static auxiliary equation in the form of Poisson's potential equation. In short, they correlated between the static and dynamic states through an auxiliary state.

The E-field and B-field originating from a system of charges and currents when the system is stationary in free space are governed by Poisson's equations; *viz.*

$$\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2=-\rho/\varepsilon_0 \tag{1}$$

and

$$\partial^2\mathbf{A}/\partial x^2+\partial^2\mathbf{A}/\partial y^2+\partial^2\mathbf{A}/\partial z^2=-\mathbf{J}/\varepsilon_0c^2 \tag{2}$$

where Φ and ρ are the scalar potential and charge density, and $\mathbf{A}$ and $\mathbf{J}$ are the vector potential and current density of the system stationary in free space, ε_0 and μ_0 are the permittivity and permeability of free space, $c=1\big/\sqrt{\mu_0e_0}$, and the introduced Cartesian coordinate system is in free space. From the above equations we have:

$$E_x=-\partial\Phi/\partial x,\ \ E_y=-\partial\Phi/\partial y,\ \ E_z=-\partial\Phi/\partial z \tag{3}$$

and

$$\mathbf{B}=\nabla\times\mathbf{A} \tag{4}$$

When this system of charges and currents is stationary in free space, we may call it 'the stationary system S_0'. But if the system moves with constant translational velocity, what will the E-field and the B-field in the free space be? We know that when this system moves in free space, the E-field in free space will induce a B-field $\mathbf{B}^*$, which, if varying, will induce an electric field. Therefore, we may say qualitatively that when the system moves, the E-field in free space changes its magnitude and direction, and an induced B-field $\mathbf{B}^*$ emerges. The same happens for the B-field: it will also change its magnitude and direction, and an induced E-field $\mathbf{E}^*$ will emerge.

Let us now proceed to deduce the exact formulas for the E-field and the B-field while the system moves in free space with constant translational velocity $\mathbf{u}$, simply using Maxwell's equations, in the way first exemplified by Oliver Heaviside, Thomson and their contemporaries. As is well known, when the system translates in free space with a constant translational velocity, the E-field and the B*-field are governed by d'Alembert's equation, *viz.*,

$$\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2-\partial^2\Phi/c^2\partial t^2=-\rho/\varepsilon_0 \tag{5}$$

and

$$\partial^2\mathbf{A}^*/\partial x^2+\partial^2\mathbf{A}^*/\partial y^2+\partial^2\mathbf{A}^*/\partial z^2-\partial^2\mathbf{A}^*/c^2\partial t^2=-\rho\mathbf{u}/\varepsilon_0c^2 \tag{6}$$

Assuming $\mathbf{u}$ in the OX direction and comparing (5) and (6), we have

$$A_x^*=u\Phi/c^2,\ \ A_y^*=0,\ \ A_z^*=0 \tag{7}$$

(u_y and u_z being zero). Here

$$E_x=-\partial\Phi/\partial x-\partial A_x^*/\partial t,\ E_y=-\partial\Phi/\partial y,\ E_z=-\partial\Phi/\partial z \tag{8}$$

(A_y^* and A_z^* being zero), and

$$\mathbf{B}^*=\nabla\times\mathbf{A}^* \tag{9}$$

We may call such a system 'the dynamic system S′', where charges (and currents) inside the system move with the system in free space with a constant translational velocity $\mathbf{u}$ in the OX direction. Here $\Phi(x,t)$ is a function of two independent variables, x and t. From the definition of two independent variables we have

$$d\Phi=(\partial\Phi/\partial x)dx+(\partial\Phi/\partial t)dt=(\partial\Phi/\partial x)u\,dt+(\partial\Phi/\partial t)dt$$

(dx being equal to $u\,dt$). Moreover, in such a situation, the potentials at the point (x,y,z) at the instant t and the potentials at the point ($x+u\,dt,\ y,\ z$) at the instant $t+dt$ in free space will be the same; *i.e.*,

$$\Phi(x,y,z,t)=\Phi(x+udt,y,z,t+dt)=\Phi(x,y,z,t)+d\Phi$$

From the above two equations we have

$$\Phi = \Phi + (\partial\Phi / \partial x)u\,dt + (\partial\Phi / \partial t)dt \tag{10}$$

$$\therefore \partial\Phi / \partial t = -u\partial\Phi / \partial x \quad \text{and} \quad \partial^2\Phi / \partial t^2 = +u^2\,\partial^2\Phi / \partial x^2 \tag{11,12}$$

Similarly, using Eqs. (7) and (11),

$$\partial A_x^* / \partial t = -(u^2 / c^2)\partial\Phi / \partial x \tag{13}$$

Now, Eq. (5) could be written as

$$(1 - u^2 / c^2)\partial^2\Phi / \partial x^2 + \partial^2\Phi / \partial y^2 + \partial^2\Phi / \partial z^2 = -\rho / \varepsilon_0 \tag{14}$$

Attach the co-ordinates x', y', z' to the moving system, and let these coincide with the co-ordinates x, y, z of the free space at $t = 0$. Now imagine an elongation of the moving system as below towards –OX, which is the direction of motion of the system:

$$x' = x\Big/\sqrt{1 - u^2 / c^2}\,,\; y' = y,\; z' = z \tag{15}$$

for which the transformation of volume charge density ρ becomes

$$\rho' = \rho k \text{ with } k = \sqrt{1 - u^2 / c^2} \tag{15a}$$

and Eq. (14) takes the form

$$\partial^2\Phi / \partial x'^2 + \partial^2\Phi / \partial y'^2 + \partial^2\Phi / \partial z'^2 = -\rho / \varepsilon_0 \tag{16}$$

which is again a Poisson equation.

If electromagnetic action is considered after the time t of the instant when the coordinate attached to the system coincided with the coordinate attached to free space, Eq. (15) should take the form

$$x' = \gamma(x - ut),\; y' = y,\; z' = z) \tag{17}$$

where

$$\gamma = 1 / k = 1\Big/\sqrt{1 - u^2 / c^2} \tag{18}$$

Eq. (17) conjointly with Eq. (18) may be called 'the auxiliary transformation equation of Heaviside and Thomson'.

Below, the value of Φ in the S system will be connected to the potential Φ' of a stationary auxiliary system S' in which all the coordinates parallel to OX have been changed in the ratio determined by Eq. (15). This transformed coordinate S' (x', y', z') system, which is obviously imaginary, will be used as a mathematical tool to correlate electromagnetic phenomena between the static S_0 and the dynamic S [6(i),6(iii),7,8,9,10]. This transformed auxiliary elongated coordinate system will be called 'Auxiliary system S''. The electrodynamic role of S' was first illustrated by Thomson.

In the Auxiliary system S', we have

$$\partial^2\Phi' / \partial x'^2 + \partial^2\Phi' / \partial y'^2 + \partial^2\Phi' / \partial z'^2 = -\frac{\rho'}{\varepsilon_0} = -\frac{\rho}{\varepsilon_0}\sqrt{1 - u^2 / c^2} \tag{19}$$

where Φ' electrostatic potential in Auxiliary system S'. By comparing (16) and (19) we have, $\Phi' = \Phi k$, or

$$\Phi = \gamma\Phi' \tag{20}$$

Therefore, [using the Eqs. (13) & (20)]

$$E_x = -\frac{\partial\Phi}{\partial x} - \frac{\partial A_x^*}{\partial t} = -\frac{\partial\Phi}{\partial x} + \frac{u^2}{c^2}\frac{\partial\Phi}{\partial x} = -\frac{\partial\Phi'}{\partial x'} = E'_x$$

$$E_y = -\partial\Phi / \partial t = -\gamma\partial\Phi' / \partial y' = \gamma E'_y \quad (A_y \text{ being } 0) \tag{21}$$

$$E_z = \gamma E'_z \quad (A_z \text{ similarly being } 0)$$

From $\mathbf{B}^* = \nabla \times \mathbf{A}^*$, we have

$$B_x^* = 0,\; B_y^* = -\,uE_z / c^2 = -\gamma uE'_z / c^2 \qquad B_z^* = uE_y / c^2 = \gamma uE'_y / c^2 \tag{22}$$

Eqs. (21) & (22) are general, and may be used to determine the electric fields and the induced magnetic fields of moving charges of any shape and size.

For the independent magnetic field originating from a system of current when the system is moving with a constant translational velocity $\mathbf{u}$, we have,

$$\partial^2 A_x / \partial x^2 + \partial^2 A_x / \partial y^2 + \partial^2 A_x / \partial z^2 - \partial^2 A_x / c^2\partial t^2 = -\rho V_x / \varepsilon_0 c^2 \tag{23}$$

and

$$\partial^2 A_y / \partial x^2 + \partial^2 A_y / \partial y^2 + \partial^2 A_y / \partial z^2 - \partial^2 A_y / c^2\partial t^2 = -\rho V_y / \varepsilon_0 c^2 \tag{24}$$

and the similar equation for the z-component. Replacing Φ by $\mathbf{A}$ in Eq. (10), Eqs. (23) and (24) could be transformed (in the way previously shown) to the following:

$$\partial^2 A_x / \partial x'^2 + \partial^2 A_x / \partial y'^2 + \partial^2 A_x / \partial z'^2 = -\rho V_x / \varepsilon_0 c^2 \tag{25}$$

and

$$\partial^2 A_y / \partial x'^2 + \partial^2 A_y / \partial y'^2 + \partial^2 A_y / \partial z'^2 = -\rho V_y / \varepsilon_0 c^2 \tag{26}$$

and the similar equation for the z' component. Here $\rho\mathbf{V}$ is the current density in the S system.

In the auxiliary system S', for line currents flowing within the moving system in any arbitrary directions, and for surface currents and volume currents flowing within the moving system in the direction of movement of the system (*i.e.*, when the magnetic field depends on the length of the current element but not on its cross section), we have

$$\frac{\partial^2 A'_x}{\partial x'^2} + \frac{\partial^2 A'_x}{\partial y'^2} + \frac{\partial A'_x}{\partial z'^2} = -\frac{\rho' V_x}{\varepsilon_0 c^2} = -\frac{\rho\,V_x}{\varepsilon_0 c^2}\sqrt{1 - u^2 / c^2} \tag{27}$$

$$\partial^2 A'_y / \partial x'^2 + \partial^2 A'_y / \partial y'^2 + \partial^2 A'_z / \partial z'^2 = -\rho V_y / \varepsilon_0 c^2 \tag{28}$$

and the similar equation for the z'-component. By comparison of (25) and (27), (26) and (28), we have,

$$A_x = \gamma A'_x,\; A_y = A'_y,\; A_z = A'_z \tag{29}$$

whence

$$B_x = \left[\partial A_z / \partial y - \partial A_y / \partial z\right] = \left[\partial A'_x / \partial y' - \partial A'_y / \partial z'\right] = B'_x$$
$$B_y = \left[\partial A_x / \partial z - \partial A_z / \partial x\right] = \left[\gamma\, \partial A'_x / \partial z - \gamma\, \partial A'_z / \partial x'\right] = \gamma B'_y \quad (30)$$
$$B_z = \left[\partial A_y / \partial x - \partial A_x / \partial y\right] = \left[\gamma\, \partial A'_y / \partial x' - \gamma\, \partial A'_x / \partial y'\right] = \gamma B'_z$$

where A'_x, A'_y and A'_z are the components of the Auxiliary magnetic potential in the Heavisidean imaginary elongated system S'. For the induced vector, we have the relation $\mathbf{E}^* = -\mathbf{u} \times \mathbf{B}$, from which we have,

$$E_x^* = 0,\ E_y^* = uB_z = \gamma u B'_z,\ E_z^* = -uB_y = -\gamma u B'_y \quad (31)$$

These equations correlate between the induced electric vector in the moving system S and the auxiliary magnetic vector in S'.

Now, if the sources of an independent electric field (originating from charges of any shape and size) and an independent magnetic field (originating from line currents flowing within the system in any arbitrary directions), move with the system at the constant translational velocity $\mathbf{u}$ in free space, then from consideration of Eqs. (21), (22), (30), and (31), we can derive the following auxiliary field equations:

$$E_x = E'_x,\ E_y = \gamma\left[E'_y + uB'_x\right],\ E_z = \gamma\left[E'_z - uB'_y\right]$$
$$B_x = B'_x,\ B_y = \gamma\left[B'_y - uE'_z / c^2\right],\ B_z = \gamma\left[B'_z + uE'_y / c^2\right] \quad (32a)$$

or

$$E'_x = E_x,\ E'_y = \gamma\left[E_y - uB_x\right],\ E_z = \gamma\left[E'_z + uB'_y\right]$$
$$B'_x = B_x,\ B'_y = \gamma\left[B_y + uE_z / c^2\right],\ B'_z = \gamma\left[B_z - uE_y / c^2\right] \quad (32b)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic fields of the system of charges and currents having the constant translational velocity $\mathbf{u}$ in free space, and $\mathbf{E}'$ and $\mathbf{B}'$ are corresponding auxiliary quantities.

Thus, we see that the electromagnetic quantities in our moving system S are not connected with the same quantities of the same system at rest (S_0). These quantities of the moving system S are connected by the equation (32a) with the corresponding quantities of the system (S') in which the co-ordinates parallel to the OX axis lying along the movement of the system have been elongated by the Eq. (15).

Those Eqs. (32) are also valid for induced electromagnetic fields when the inductor or the inducted body moves with respect to free space. For dealing with steadily moving point charges, we may use the following two corollaries, the first of which correlates between the static system S_0 and the auxiliary system S', and the second of which describes the validity of Maxwell's equation for Heaviside's fields; *viz.*,

Corollary 1. In a stationary system S_0, if $\mathbf{E}_0$ is the source of $\mathbf{B}_0$, and $E_0 = bB_0$, where b is constant, then in the auxiliary system S', $E' = bB'$.

Corollary 2. It can easily be shown that Heaviside's fields obey Maxwell's equations just like Coulomb's fields do. Therefore, if a stationary dipole radiates in free space, it will also radiate while moving in free space at constant translational velocity.

3. Heaviside's Electrodynamics in Free Space

The first free-space application deals with the electric field $\mathbf{E}$ and the induced magnetic field $\mathbf{B}^*$ of a point charge Q moving with a constant translational velocity $\mathbf{u}$. Following Heaviside, the fields could be calculated at a point $P\ (x, y, z)$ in free space (considering at a particular instant, the point charge as origin and OX as the direction of motion of the charge) in the following way:

$$E'_x = \frac{Qx'}{4\pi e_0 r'^3},\ E'_y = \frac{Qy'}{4\pi e_0 r'^3},\ E'_z = \frac{Qz'}{4\pi e_0 r'^3} \quad (33)$$

where E'_x, E'_y, and E'_z are the components of auxiliary electric field in the Heavisidean auxiliary system S' at $P'\ (x', y', z')$

Now, the distance of the point P' from the origin in this S' system, $r' = \sqrt{x'^2 + y'^2 + z'^2}$ where $P'\ (x', y', z')$ of the S' system is the corresponding point $P\ (x, y, z)$ of the S system. The Coordinate of the corresponding point P is (x, y, z) in Cartesian System and (r, θ, φ) in spherical polar coordinate where r is the distance from the origin, θ is the angle down from the x axis and φ is the angle around from the z axis. Therefore, $x = r\cos\theta$, $y = r\sin\theta\sin\varphi$, $z = r\sin\theta\cos\varphi$, from which $x^2 = r^2\cos^2\theta$ and $y^2 + z^2 = r^2\sin^2\theta$. Therefore:

$$\begin{aligned} r'^3 &= (x'^2 + y'^2 + z'^2)^{3/2} \\ &= (\gamma^2 x^2 + y^2 + z^2)^{3/2} \text{ [using Eq. (15)]} \\ &= (\gamma^2 r^2 \cos^2\theta + r^2\sin^2\theta)^{3/2} \text{ [in polar coordinates]} \\ &= \gamma^3 r^3 \left[1 - (u^2/c^2)\sin^2\theta\right]^{3/2} \text{ [a result independent of } \varphi] \end{aligned}$$

$$\begin{aligned} \text{Now, } E &= \sqrt{E_x^2 + E_y^2 + E_z^2} \\ &= \sqrt{E_x'^2 + \gamma^2 E_y'^2 + \gamma^2 E_z'^2} \text{ [using Eq. (21)]} \\ &= \frac{Q}{4\pi\varepsilon_0 r'^3}\sqrt{x'^2 + \gamma^2 y'^2 + \gamma^2 z'^2} \text{ [using Eq. (33)]} \\ &= \frac{Q}{4\pi e_0 r'^3}\gamma r \text{ [using Eq. (15)]} \end{aligned}$$

Thus, independent of φ, we have:

$$\begin{aligned} E &= \frac{Q}{4\pi\varepsilon_0}\frac{\gamma r}{\gamma^3 r^3}\left[1 - (u^2/c^2)\sin^2\theta\right]^{-3/2} \\ &= \frac{Qk^2}{4\pi e_0 r^2}\left[1 - (u^2/c^2)\sin^2\theta\right]^{-3/2} \end{aligned} \quad (34)$$

The auxiliary $\mathbf{E}'$ is directed along OP'. Therefore, the real field $\mathbf{E}$ is directed along OP.

Now, from $\mathbf{B}^* = \nabla \times \mathbf{A}^*$, we have,

$$B_x^* = 0,\ B_y^* = -\frac{u}{c^2}E_z = -\gamma\frac{u}{c^2}E'_x,\ B_z^* = \frac{u}{c^2}E_y = \gamma\frac{u}{c^2}E'_y \quad (35)$$

from which the induced magnetic field is

$$\mathbf{B}^* = \mathbf{u} \times \mathbf{E}/c^2 \quad (36)$$

With a little analysis, it can be shown that Eq. (34) and (36) are the same for a charged ellipsoid having its axes with ratios k:1:1 moving with a constant translational velocity $\mathbf{u}$ in free space, k being in the direction of motion of the ellipsoid. Thus Oliver Heaviside (1850 - 1925) the greatest electromagnetician after Maxwell (1831- 1879) has shown that a charged ellipsoid having its axes with ratios k:1:1 while moving with a constant translational velocity $\mathbf{u}$ in free space produces the same external effect as that of a similarly moving point charge [9,10], k acting in the direction of motion of the charge.

Electromagnetic momentum

The electromagnetic momentum of a Heaviside's ellipsoid (with the axes δRk: δR: δR) while moving rectilinearly with a velocity $\mathbf{u}$ in the OX direction in free space is

$$P_x = \int \left(D_y B_x^* - D_z B_y^* \right) d\tau = \frac{u}{c^2}\varepsilon_0 \int \left(\gamma^2 E_y'^2 + \gamma^2 E_z'^2 \right) k\, d\tau'$$

$$= \gamma \frac{u}{c^2}\varepsilon_0 \int \left(E_y'^2 + E_z'^2 \right) d\tau' \qquad cf.\ \text{Eqs. (21)\&(22)}$$

where $d\tau$ is the volume element in the S system and $d\tau'$ is the corresponding volume element in the S′ system. Here in the S′ system Heaviside's ellipsoid is exactly a sphere. Therefore, when evaluated between δR and ∞, $\int E'^2 d\tau' = q^2 / 4\pi\varepsilon_0^2 \delta R$, and each integral in the previous equation is equal to $q^2 / 12\pi\varepsilon_0^2 \delta R$. Therefore,

$$P_x = q^2 \mathbf{u} / 6\pi\varepsilon_0 c^2 \delta R k = m\mathbf{u} \tag{37}$$

which is the electromagnetic momentum of a point charge moving rectilinearly in free space with velocity $\mathbf{u}$, where m_0 and m are the electromagnetic masses of the point charge moving with the velocities near zero and $\mathbf{u}$ respectively in free space such that

$$q^2 / 6\pi\varepsilon_0 c^2 \delta R = m_0 \text{ and } m_0 / k = m \tag{38}$$

(Consult Hajra 1998 [11] and Searle 1897 [12] for an alternative deduction of electromagnetic momentum of a steadily moving point charge.)

Electromagnetic force acting on a point charge moving in free space

a) in a direction parallel to the direction of the uniform electric field operating in free space,

$$F_{||} = (dP / du) a_{||} = (m_0 / k^3) a_{||} \tag{39}$$

where $a_{||}$ is the acceleration of the point charge in the direction parallel to the field.

b) at a direction perpendicular to the direction of the uniform electric field operating in free space.

$$F_\perp = (|\mathbf{P}| / |\mathbf{u}|) a_\perp = (m_0 / k) a_\perp \tag{40}$$

where $a_\perp$ is the acceleration of the point charge in the direction perpendicular to the field.

(A general treatment will show $\mathbf{F} = m\, d\mathbf{u}/dt + \mathbf{u}(\mathbf{F}\cdot\mathbf{u}) / c^2$.)

Similarly, evaluated between δR and ∞, the energy of a point charge having a constant translational velocity in free space is

$$\mathcal{E} = \int (\partial P / \partial t) dx = m_0 c^2 / k = mc^2 \tag{41}$$

Frequencies of light emitted from a source having a constant translational velocity in free space -

Let an electric force $\mathbf{F}_0$ (originating from a small charge) drive a point charge back and forth from one end to the other end of a radiating dipole stationary in free space. Then,

$$\mathbf{F}_0 = -m_0 \omega_0^2 \mathbf{S} \tag{42}$$

the velocity of oscillation being small, where m_0 is the electromagnetic mass of the charge in the stationary dipole, ω_0 is the radian frequency of oscillation of the charge, S is the separating distance of the dipole.

Now, if the dipole moves with a velocity $\mathbf{u}$ in free space in any direction perpendicular to its direction of oscillations, the electric force and the magnetic force acting on the charge will be respectively from Eqs. (34) and (36), (when $\theta = 90^0$) γF_0 and $(u^2 / c^2)\gamma F_0$. Therefore, total electromagnetic force acting on the moving charge is

$$\mathbf{F} = \gamma \mathbf{F}_0 - (u^2 / c^2)\gamma \mathbf{F}_0 = \mathbf{F}_0 k \tag{43}$$

Now, under the circumstance that the dipole moves and radiates, we have

$$\mathbf{F} = -m\omega^2 \mathbf{S} \tag{44}$$

where m is the electromagnetic mass of the charge in the moving dipole, ω is the frequency of oscillation of the charge which is moving with a velocity $\mathbf{u}$ in free space with the dipole and $\mathbf{F}$ is the electromagnetic force acting on the moving charge.

From Eqs. (40), (42), (43) and (44) for the dipole moving with an uniform velocity in any direction perpendicular to its direction of motion we have,

$$\omega = \omega_0 k \tag{45}$$

Now, if that radiating dipole while moving with a velocity $\mathbf{u}$ towards a direction parallel to OX, is seen from the origin at any point P which makes an angle θ with OX axis at the origin, we have from Doppler

$$\omega_{\text{observed}} = \frac{\omega_0 k}{1 + (u / c)\cos\theta} \tag{46}$$

whence

$$\omega_{\text{trans.}} = \omega_0 k \tag{47}$$

i.e., the well known transverse Doppler effect, if the dipole moving in a direction parallel to OX and oscillating in a direction parallel to OZ is seen at a point $(0, y, 0)$ from the origin.

The period of oscillation (t) for a radiating dipole having a constant translational velocity in free space

For a dipole stationary in free space,

$$t_0 = 2\pi / \omega_0 \tag{48}$$

where t_0 is the oscillation period and ω_0 is the radian frequency. If the same radiating dipole moves with a velocity $\mathbf{u}$ in free space, then for the moving dipole the oscillation period t and radian frequency ω satisfy

$$t = 2\pi / \omega \tag{49}$$

Comparing Eqs. (48) and (49) with the Eq. (45) we have

$$t = \gamma\, t_0 \tag{50}$$

or the period of oscillation of a moving dipole increases with its velocity in free space.

Life spans of radioactive particles having a constant translational velocity in free space

a) Proton-proton decay or electron-electron Decay

Consider two similar point charges tied by some unknown forces. The repelling electric force is here tending to destroy the equilibrium whereas the unknown forces are keeping the charges tied together. Therefore, spontaneous transformation of those particles should depend also on the repulsive electromagnetic force, just as on time.

We see that the number of radioactive particles of one particular species decreases with time and the slowing down of the velocity of the particles. The decrease obeys a certain law that we would like to find.

Let at the initial instant of $t = 0$, there be N_0 radioactive particles of a particular species. Let us find the number N of those particles that will remain untransformed by an arbitrary time t. Since we are dealing with spontaneous transformation, we may presume that the rate at which the total quantity N of radioactive particles is diminishing at any instant is **i)** proportional to the total quantity N of radioactive particles present at that instant when the electromagnetic force F acting inside the particles is constant, and **ii)** proportional to the electromagnetic force F acting inside the particles when N is constant, which may be *a priori* plausible. Therefore we may write $dN/dt = -\lambda FN$ where F and N both vary. Moreover, we have, $N = N_0 f(F, t)$.

In the circumstances where F is constant (*i.e.*, where the radioactive particles either at rest or at uniform motion of translation in free space), combining above two equations we have,

$$N = N_0 e^{-\lambda F t} \tag{51}$$

where λ is the proportionality constant with dimensions of Newton^{-1} Second^{-1}.

Now, if N_0 radioactive particles of similarly charged bodies are at rest in free space, and if we have N untransformed particles after the time t_0, then we have,

$$N = N_0 e^{-\lambda F_0 t_0} \tag{52}$$

where F_0 is the repelling force acting on the charged particles at rest in free space.

Now if the charged particles move with a velocity $\mathbf{u}$ in free space in any direction perpendicular to their direction of attachment, after a time t we will find N untransformed particles such that

$$N = N_0 e^{-\lambda F t} \tag{53}$$

comparing Eqs. (52) and (53) with Eq. (43), we have

$$t = \gamma\, t_0 \tag{54}$$

b) Positive point charge - negative point charge decay

Consider some unknown forces separate two dissimilar point charges. The attracting electric force is trying to destroy the equilibrium, whereas the unknown forces are keeping the charges separated. By the similar arguments as in **a)** we have $t = \gamma\, t_0$

c) Similarly for negative or positive muon-type decay

We may consider that a muon is a point negative or positive charge tied with a point mass by the attractive electric force that is giving stability, but some other unknown repelling forces responsible for decay are acting here to destabilize the equilibrium. So here,

$$N = N_0 e^{-\lambda t / F} \tag{55}$$

as N decreases with t but increases with F.

Now, the decay equations of muons for stationary and moving states respectively could be written as follows:

$$N = N_0 e^{-\lambda t_0 / F_0} \tag{56}$$

$$N = N_0 e^{-\lambda t / F} \tag{57}$$

The magnetic force is here 0, because the magnetic field is non-operative on moving mass, only the electric force is here

$$\mathbf{F} = \gamma\, \mathbf{F}_0 \;\; [\textit{cf.}\ \text{Eq. (43)}] \tag{58}$$

Comparing Eqs. (56), (57) and (58), we may write for muon decay

$$t = \gamma\, t_0 \tag{59}$$

Thus, we may conclude that the lifetime of a charged electromagnetic radioactive particle increases with its velocity in free space.

We have deduced all those equations from Maxwell's field equations using the Eq. (43), which is the relation of electromagnetic force acting on a point charge in the steadily moving system and in the stationary system. Now, from the consideration of Maxwell, all those phenomena, like transverse Doppler's effect and increment of life spans of steadily moving radioactive particles, are possible only when the radiating dipoles and radioactive particles move steadily in free space and a stationary observer measures the effects. In cases when the radiating dipoles and radioactive particles are at rest in free space and the observer measures the effect while moving steadily, according to Maxwell, no transverse Doppler's effect and no increment of life spans of radioactive particles are possible. But, according to special relativity, in both the cases, transverse Doppler's effect and increments of life spans of radioactive particles should be observed. Therefore, if the electromagnetic sources are stationary in free space and the observer while moving finds such effects, only then special relativity could be held superior to electrodynamics. But, these were never gone into.

Velocity of light in a medium moving in free space

We know from Maxwell that the relation between the electric field $\mathbf{E}_0$ and the magnetic field $\mathbf{B}_0$ in a ray moving in OX direction inside a dielectric at rest in free space is

$$E_{0y} / B_{0z} = c / n \tag{60}$$

where n is the refractive index of the medium.

Now, if the dielectric moves with a velocity $\mathbf{u}$ in the OX direction in free space, V_x is the velocity of the ray in OX direction with respect to free space and E_y and B_z are the electric field and the magnetic field of the ray in the moving dielectric, we have,

$$V_x = \frac{E_y}{B_z} = \frac{\gamma(E'_y + uB'_x)}{\gamma[B'_x + (u/c^2)E'_y]} \qquad [\textit{cf.}\ \text{Eq. (32a)}]$$

Dividing the numerator and the denominator by B'_z and using the Corollary 1 of Section 2, *i.e.*, if $E_{0y} / B_{0z} = c/n$, then $E'_y / B'_z = c/n$, in the case where $\mathbf{E}_0$ is the source of $\mathbf{B}_0$, we have

$$V_x = \frac{c/n + u}{1 + u/nc} \approx \frac{c}{n} + u(1 - 1/n^2) \tag{61}$$

Velocity of charges in a conductor moving in free space

Suppose that a point charge q is moving with speed v_x in the OX direction inside the surface of a conductor (at rest in free space) wherein an electric field $\mathbf{E}_0$ and a magnetic field $\mathbf{B}_0$ are operating on the charge. As the charge is flowing in the OX direction, we may presume that the electromagnetic force acting on the charge towards OY directions $F_{0y} = 0$. Now from Lorentz,

$$F_{0y} = q\,(E_{0y} - v_x B_{0z}) \tag{62}$$

Combining those two equations given above, we have $q\,(E_{0y} - v_x B_{0z}) = 0$. Therefore,

$$v_x = E_{0y} \big/ B_{0z} \tag{63}$$

Now, if the conductor moves with a velocity $\mathbf{u}$ in the OX direction in free space, the electric field and magnetic field inside the conductor change from $\mathbf{E}_0$ and $\mathbf{B}_0$ to $\mathbf{E}$ and $\mathbf{B}$ and therefore, the velocity of the charge changes from $\mathbf{v}$ to $\mathbf{V}$ inside the conductor. From similar analyses as given in framing the Eq. (63), we have for the velocity component V_x of the charge (with respect to free space) in the moving conductor in terms of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$, [*cf.* Eqs. (32a)]

$$V_x = \frac{E_y}{B_z} = \frac{\gamma(E'_y + uB'_z)}{\gamma(B'_z + uE'_y / c^2)} \tag{64}$$

where E_y and B_z are electric and magnetic field components in the moving conductor and E'_y and B'_z are the similar components in the auxiliary system. Now if $E_{0y} / B_{0x} = v_x$ and $\mathbf{E}_0$ is the source of $\mathbf{B}_0$, then from Corollary 1 of Section 2 we have, $E'_y / B'_z = v_\text{x}$.

Dividing the numerator and denominator of Eq. (64) by B'_z, and using the corollary 1 of Section 2 [Electrodynamics of Heaviside and Thomson] as given above, we have,

$$V_x = \frac{u + v_x}{1 + uv_x / c^2} \tag{65}$$

When the charges moves in any arbitrary direction in XY plane which is the surface of the conductor, we have (say),

$$E_{0y} / B_{0x} = v_x \text{ and } E_{0z} / B_{0z} = v_y \tag{66, 67}$$

Now, if the plane moves with a velocity $\mathbf{u}$ in free space in the OX direction

$$V_y = \frac{E_x}{B_z} = \frac{E'_x}{\gamma\left[B'_z + (u/c^2)E'_y\right]} = \frac{kE'_x / B'_z}{1 + (u/c^2)\,E'_y \big/ B'_z}$$

or (when $\mathbf{E}_0$ is the source of $\mathbf{B}_0$)

$$V_y = v_y k \big/ (1 + uv_x / c^2) \quad \text{and} \quad V_z = v_z k \big/ (1 + uv_x / c^2) \tag{68, 69}$$

From (65), (68) and (69) we have,

$$c^2 - V^2 = c^2(c^2 - u^2)(c^2 - v^2) \big/ (c^2 + uv_x)^2 \tag{70}$$

Charge moving inside a conductor that moves in free space

Suppose that some point charges are moving inside a conductor with a velocity υ while the conductor is at rest in free space. So, the electromagnetic mass of a moving point charge q is $m = m_0 \big/ \sqrt{1 - v^2 / c^2}$, its electromagnetic momentum is $p = m_0 v \big/ \sqrt{1 - v^2 / c^2}$, its energy is $\varepsilon = m_0 c^2 \big/ \sqrt{1 - v^2 / c^2}$, and the electromagnetic force acting on the point charge is $\mathbf{f} = q(\mathbf{E}_0 + \mathbf{v} \times \mathbf{B}_0)$. Now if the conductor moves with a velocity $\mathbf{u}$ in free space, for the same point charge we have:

Electromagnetic mass [using Eq. (70)]

$$M = m_0 \big/ \sqrt{1 - V^2 / c^2} = m(1 + u\upsilon_x / c^2) \big/ \sqrt{1 - u^2 / c^2} \tag{71}$$

Electromagnetic momentum [using Eqs. (65), (68). & (70)]

$$P_x = \frac{m_0 V_x}{\sqrt{1 - V^2 / c^2}} = \frac{p_x + \varepsilon u / c^2}{\sqrt{1 - u^2 / c^2}},\ P_y = p_y,\ P_z = p_z \tag{72}$$

Energy

$$\mathcal{E} = m_0 c^2 \big/ \sqrt{1 - V^2 / c^2} = (\varepsilon + up_x) \big/ \sqrt{1 - u^2 / c^2} \tag{73}$$

Force acting on a point charge [using Eqs. (32a), (65), (68). (69), (70), & (71)]

$$F_x = q\left[E_x + (\mathbf{V} \times \mathbf{B})_x\right] = \frac{f'_x + u(\mathbf{v} \cdot \mathbf{f}')/c^2}{1 + uv_x / c^2}, \quad F_y = \frac{f'_y k}{1 + uv_x / c^2},\ F_z = \frac{f'_z k}{1 + uv_x / c^2} \tag{74}$$

where $$\mathbf{f}' = q[\mathbf{E}' + \mathbf{v} \times \mathbf{B}'] \tag{75}$$

gives the auxiliary force. In the special case $E_{0x} = 0$, $E_{0y} = E'_y$ and $E_{0z} = E'_z$, we have

$$F_x = \frac{f_x + u(\mathbf{v} \cdot \mathbf{f}) / c^2}{1 + uv_x / c^2}, \; F_y = \frac{f_y k}{1 + uv_x / c^2}, \; F_z = \frac{f_z k}{1 + uv_x / c^2} \quad (76)$$

4. Auxiliary Transformation Equations

This Section derives Lorentz's auxiliary equations from those of Heaviside. Following Heaviside and Thomson, Lorentz engaged himself in developing some new auxiliary transformation equations. He liked to solve optical problems of moving bodies as well as electrodynamic problems of charges moving with low and high velocities [6(ii)] through those transformation equations. These transformation equations should reduce the equations of moving systems to the form of ordinary formula that hold for systems at rest.

The problem Heaviside and Thomson addressed was to model the potentials for charges having a constant translational velocity in free space. They solved this problem by transforming the d' Alembert's equation in an invariant form with Poisson's in the auxiliary system. Lorentz's problem was to model the radiation from moving bodies. He solved this problem by transforming Maxwell's equations for the moving radiating body to the same form for the auxiliary system, which correlates between the static and dynamic radiation states.

By dint of Corollary 2 of Section 2, we have,

$$\partial^2 \mathbf{E} / \partial x^2 + \partial^2 \mathbf{E} / \partial y^2 + \partial^2 \mathbf{E} / \partial z^2 - \frac{1}{c^2} \partial^2 \mathbf{E} / \partial t^2 = 0 \quad (77)$$

where $\mathbf{E}$ is the electric field of a radiating dipole moving with a constant translational velocity $\mathbf{u}$ in free space. From this we get

$$x^2 + y^2 + z^2 = c^2 t^2 \quad (77a)$$

To solve radiation problems in a way analogous to that shown in Section 2, we are to keep the Maxwell's equation in the same form in the S' system; *i.e.*, it is now required that

$$\partial^2 \mathbf{E}' / \partial x'^2 + \partial^2 \mathbf{E}' / \partial y'^2 + \partial^2 \mathbf{E}' / \partial z'^2 - \frac{1}{\mathbf{c}^2} \partial^2 \mathbf{E}' / \partial t'^2 = 0 \quad (78)$$

where $\mathbf{E}'$ is the auxiliary electric field of Heaviside and Thomson in the S' system. That is,

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \quad (78a)$$

Subtracting Eq. (77a) from Eq. (78a) and using the Heaviside-Thomson auxiliary equations

$$x' = \gamma(x - ut), \; y' = y, \; z' = z \quad (79)$$

to replace the primed variables, we get

$$c^2 t'^2 = (x - ut)^2 / (1 - u^2 / c^2) - x^2 - c^2 t^2$$

or

$$t' = \gamma(t - ux / c^2) \quad (80)$$

the famous auxiliary time equation of Lorentz.

An interesting fact about the equations is that the inverse Lorentz Transformation equations (which could be deduced from Lorentz Transformation equations) have the same form as the Lorentz Transformation equations themselves; *i.e.*,

if $\quad x' = \gamma(x - ut), \; y' = y, \; z' = z, \; t' = \gamma(t - ux / c^2) \quad$ (81a-d)

then $\quad x = \gamma(x' + ut'), \; y = y', \; z = z', \; t = \gamma(t' + ux' / c^2) \quad$ (82a-d)

Therefore, Lorentz transformation equations can be deduced in reverse from the dyads of Eqs. (77a) & (78a) and (81a-c), (82a-c). (These transformation equations were later observed and used by A. Einstein in his theory).

It should be mentioned here that all the quantities x', y', z', t' $\mathbf{E}', \mathbf{B}'$, *etc.*, are auxiliary, so Eq. (78) is auxiliary. Thus, from the standpoint of electrodynamics, Lorentz Transformation Equations are tactical, just like the tactical equations of Heaviside and Thomson. These auxiliary equations are very useful to solve correctly the radiation problems associated with steadily moving electromagnretic bodies. The auxiliary equations of Heaviside and Thomson (79) are general, but Lorentz's auxiliary time Eq. (80) is not general. It is only applicable in radiation problems of moving systems and in some special cases. To consider Lorentz's auxiliary time equations as general is contrary to the principles of electrodynamics, and to consider all four Lorentz's transformation Eqs. (79) & (80) as 'real' is an over simplification of electrodynamics and of nature.

5. Conclusion

We see that the electrodynamic equations, including auxiliary Lorentz's transformation equations, can easily be deduced from Maxwell's field equations following the ingenious mathematical treatment initiated by Heaviside and Thomson for the study of a steadily moving system of charges. We present all the above simple physical derivations as the first step toward an ultimate 'collapse of Special Relativity Theory', to be developed further in subsequent publication(s).

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