ECE 317
Summer 2005
Lab Assignment #3

Matlab Analysis of Time and Frequency Response of LTI Systems

1) Consider an LTI system described by a first order differential equation,

   $a_0 \frac{dy(t)}{dt} + y(t) = 4x(t)$, with zero initial conditions.

   A) Write the equivalent rational transfer function for this system, H(s).

   B) Find the step response of the system, $y_s(t)$, when a_0 = 0.1, 0.4, 0.8, and 1.5. Plot all four responses for at least 10 seconds on the same plot. Use a legend and make sure that each line is distinct.

   C) Based on the previous part, make a table that includes the steady-state value and the time it takes to reach steady state (within 1%) for each value of a_0.

2) Consider the system given in problem #1 with $a_0 = \frac{1}{40\pi}$ and an input

   $x(t) = A\cdot\cos(2\pi ft)\cdot u(t)$, where A = 1.

   A) Find the response of the system, y(t), when f = 1, 10, 20, 40, and 80 Hz. Create five plots in which the input and corresponding output are on the same plot. Use legends and make sure that each line is distinct. Each plot should cover exactly 5 periods of the input signal. For larger values of f, make sure your time step is small enough (e.g. 0.0002 sec) to provide smooth lines in your plots.

   B) Based on the plots in the previous part, make a table that includes the amplitude and phase of the output signal at steady state for each frequency. **<u>Hint</u>**: At steady state, the output can be written as $y_{ss}(t) = B\cdot\cos(2\pi ft + \theta)\cdot u(t)$, where B is the amplitude and θ is the phase shift (in radians). Note that B and θ are functions of the frequency f.

   C) Based on the previous part, plot the magnitude response (B/A versus frequency) and phase response (θ versus frequency) of the system.

D) What type of filter does this system represent (low-pass, high-pass, etc.)? Why?

3) Consider an LTI system described by a second order differential equation,

$$a_0 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2 y(t) = b_0 \frac{d^2 x(t)}{dt^2} + b_1 \frac{dx(t)}{dt} + b_2 x(t)$$, with zero initial conditions.

A) Find the step response of the system, $y_1(t)$, when $a_0 = 1$, $a_1 = 0.8$, $a_2 = 1$, $b_0 = b_1 = 0$ and $b_2 = 2$. Plot at least 15 seconds of the response.

B) Based on the previous part, find the peak value and final (steady-state) value.

C) Find the step response of the system, $y_2(t)$, when the same coefficients are used as before except $a_1 = 1.6$. Plot at least 10 seconds of the response.

D) Based on the previous part, find the peak value and final (steady-state) value.

E) Did the peak value differ between parts A/B and C/D? Explain.

4) Consider the system given in problem #3 with $a_0 = 1$, $a_1 = (60\pi)\sqrt{2}$, $a_2 = (60\pi)^2$, $b_0 = 1$, $b_1 = 0$, and $b_2 = 0$.

A) Write the equivalent rational transfer function for this system, H(s).

B) Using the freqs() command, plot the magnitude and phase responses of this system for $0 \le f \le 100$ Hz. Be sure to label your plots.

C) What type of filter does this system represent? Why?

D) Determine the cutoff frequency (in Hz), at which the magnitude response is 0.707 times its largest value.

5) Consider the system given in problem #3 with $a_0 = 1$, $a_1 = (50\pi)(0.4)$, $a_2 = (50\pi)^2$, $b_0 = 0$, $b_1 = a_1$, and $b_2 = 0$.

A) Write the equivalent rational transfer function for this system, H(s).

B) Using the freqs() command, plot the magnitude and phase responses of this system for $0 \le f \le 80$ Hz. Be sure to label your plots.

C) What type of filter does this system represent? Why?

D) Determine the center frequency and bandwidth of the filter (in Hz).