ECE 317
Summer 2005
Review Questions for Exam #1

General topics you should know for Exam #1:

a) Signals
   i) Basic signals
   ii) Signal operations
   iii) Signal properties
b) Systems
   i) System properties
   ii) Linear time invariant (LTI) systems
      - Determining system response via convolution
      - Determining system properties from the impulse response
c) Laplace transform
   i) Evaluating the Laplace transform integral
   ii) Region of convergence (ROC)
   iii) Properties of the Laplace transform
   iv) Initial and final value theorems
d) Inverse Laplace transform
   i) Partial fraction expansion (PFE)
   ii) Solving LTI systems with the Laplace transform

<u>Questions:</u>

1) Consider the signal: $x(t) = P_2(t/2)$
   A) Is x(t) odd, even, or neither?
   B) Is x(t) or finite or infinite duration?
   C) Is x(t) causal or noncausal?
   D) Find the energy and normalized average power of x(t). Is x(t) an energy signal, a power signal, or neither?

2) Consider the composite signal:
   $x(t) = 2r(t+2)P_6(t+2) - 3u(t) + 4[r(t-1) - r(t)] + [2 + \cos(\pi(t-2))]u(t-2)$
   A) Sketch x(t) for $-2 \le t \le 4$.

3) Consider a system with the following output equation: $y(t) = 0.1 + x^{1/2}(t)$
   A) What is the equation for the impulse response, h(t)?
   B) Is the system linear?
   C) Is the system time invariant?
   D) Is the system causal?
   E) Is the system stable?
   F) Is the system memoryless?

4) Consider an LTI system with an impulse response of h(t) = u(t)
   A) Is the system stable?
   B) Find the step response via convolution.
   C) Use the previous result to find the system's response to P_2(t–1).

5) Using the Laplace transform integral, find the Laplace transform and ROC for $3P_2$(t–1).

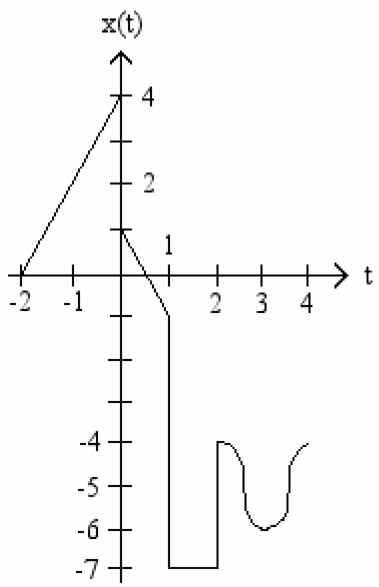
6) Using a Laplace transform table (pg. 350 in the book) and properties of the Laplace transform, determine the Laplace transform and ROC for:
   x(t) = r(t/2) + (t–1)u(t–1) – 3u(t)*cos(t)u(t)
   Note: The '*' indicates convolution.

7) Given: $X(s) = \dfrac{6s^2+1}{2s^2+4s}$
   A) Find the partial fraction expansion of $X(s)$.
   B) Find the inverse Laplace transform of $X(s)$.

Solutions:

1) $x(t) = P_2(t/2) = P_4(t)$
   A) Even
   B) Finite duration
   C) Noncausal
   D) x(t) is purely real and positive, so $|x(t)|^2 = x^2(t)$
      $E = 4$ J, $P_{avg} = 0$ W
      x(t) is an energy signal

2)

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t = [-2 : 0.01 : 4];
r1 = zeros(size(t));
r1(t >= -2) = (t(t >= -2) + 2);
p1 = zeros(size(t));
p1(t >= -5 & t <= 1) = 1;
u1 = zeros(size(t));
u1(t >= 0) = 1;
r2 = zeros(size(t));
r2(t >= 1) = (t(t >= 1) - 1);
r3 = zeros(size(t));
r3(t >= 0) = t(t >= 0);
c1 = zeros(size(t));
c1 = cos(pi*(t-2));
u2 = zeros(size(t));
u2(t >= 2) = 1;
x = 2*r1.*p1 - 3*u1 + 4*(r2 - r3) + (2 + c1).*u2;
figure;
plot(t, x);
grid on;
title('Problem #2');
ylabel('Amplitude');
xlabel('Time');
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3) $y(t) = 0.1 + x^{1/2}(t)$
   A) $h(t) = 0.1 + \delta^{1/2}(t)$
   B) Not linear (due to the exponent)
   C) Time invariant
   D) Causal (doesn't depend on future input)
   E) Stable (bounded input will produce a bounded output)
   F) Memoryless (only depends on x(t), where t = the current time)

4) LTI system with h(t) = u(t)

A) Not stable: $\int_{-\infty}^{\infty}|h(t)|dt = \int_{0}^{\infty}1dt = \infty$

B) $y_s(t) = r(t)$

C) y(t) = r(t) – r(t–2)

- Let x(t) = u(t), which produces $y_s(t) = r(t)$
  We know that $P_2(t–1)$ = u(t) – u(t–2) = x(t) – x(t–2)
  Since the system is linear and time invariant,
  x(t) – x(t–2) produces an output of $y_s(t) – y_s(t–2)$ = r(t) – r(t–2)

5) $X(s) = 3\frac{1}{s} - 3e^{-2s}\frac{1}{s} = \frac{3}{s}(1-e^{-2s})$ with ROC = {all s} since $P_2(t–1)$ has a finite duration

6) $X(s) = \frac{1}{0.5}\frac{1}{(s/0.5)^2} + e^{-s}\frac{1}{s^2} - 3\frac{1}{s}\frac{s}{s^2+1^2} = 0.5\frac{1}{s^2} + e^{-s}\frac{1}{s^2} - 3\frac{1}{s^2+1}$ with ROC = {σ > 0}

- r(t/2) $\xleftrightarrow{LT}$ $\frac{1}{0.5}\frac{1}{(s/0.5)^2}$ uses the time-scaling property with a = 0.5
  ROC’ = {σ > 0·(0.5)} = {σ > 0}
- (t–1)u(t–1) = (t)u(t) shifted to the right by 1 second
  (t)u(t) $\xleftrightarrow{LT}$ $(-1)^1\frac{d^1}{ds^1}\left[\frac{1}{s}\right] = \frac{1}{s^2}$ by the multiplication by t property
  ROC’ = ROC = {σ > 0}
  (t–1)u(t–1) $\xleftrightarrow{LT}$ $e^{-1s}\frac{1}{s^2}$ by the time-shifting property
  ROC’ = ROC = {σ > 0}
- –3u(t)*cos(t)u(t) $\xleftrightarrow{LT}$ –3U(s)C(s) = $-3\frac{1}{s}\frac{s}{s^2+1^2}$ by the linearity and convolution properties
  ROC’ = $ROC_{U(s)} \cap ROC_{C(s)}$ = {σ > 0} ∩ {σ > 0} = {σ > 0}
- X(s) = the sum of each transform by the linearity property
  ROC’ = {σ > 0} ∩ {σ > 0} ∩ {σ > 0} = {σ > 0}

7)

A) $X(s) = 3 + \frac{0.25}{s} + \frac{-6.25}{s+2}$

B) $x(t) = 3\delta(t) + 0.25u(t) - 6.25e^{-2t}u(t)$