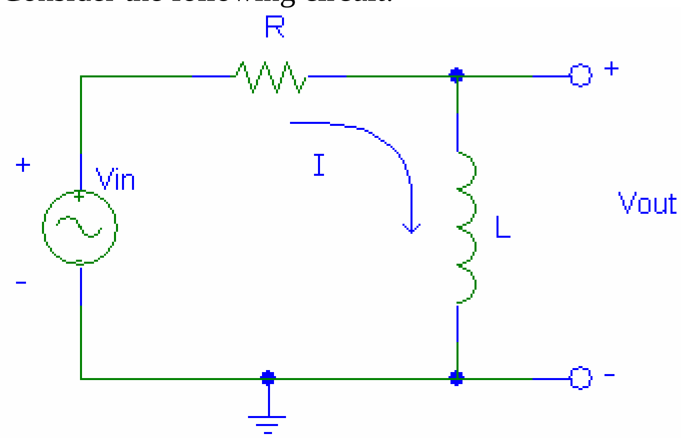
ECE 317
Summer 2005
Review Questions for Exam #2

General topics you should know for Exam #2:

a) Circuit analysis using the Laplace transform
   i) Without initial conditions
   ii) With initial conditions
b) Frequency response
   i) Magnitude response
   ii) Phase response
   iii) Steady-state response to sinusoidal inputs
c) Filters
   i) Standard filter types
   ii) Passive filter design

Questions:

1) Consider the following circuit:



Assume that $V_{in}(t) = u(t)$, R = 4 Ω, L = 2 H, and the initial current flowing through the inductor is 1 A.

A) Determine and sketch i(t).
B) Determine and sketch $V_{out}(t)$.

2) Consider an LTI system with $H(s) = \frac{s}{s+3}$.

A) Determine and sketch the step response.
B) Determine and sketch the steady-state response to an input $x(t) = 3\cos(2t + \pi/2)u(t)$.

3) Consider a composite signal $x(t) = x_1(t) + x_2(t) + x_3(t)$, where $x_1(t) = 1.5\cos(250\pi t)u(t)$, $x_2(t) = \cos(500\pi t + \pi/4)u(t)$, and $x_3(t) = 0.5\cos(750\pi t)u(t)$. Assume that only a 4.20 μF capacitor is available.

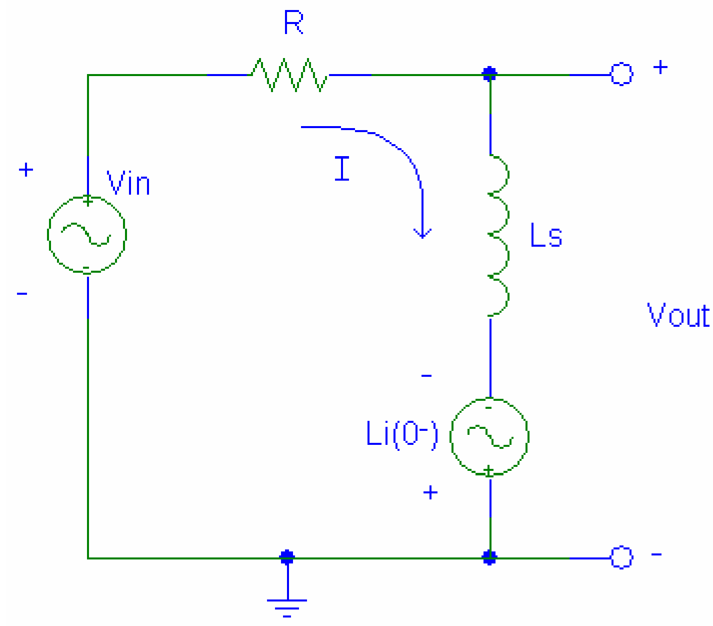
A) Design a passive second order filter to eliminate $x_2(t)$ and $x_3(t)$.

B) Design a passive second order filter to eliminate $x_1(t)$ and $x_3(t)$.

Solutions:

1)

A) $V_{in}(t) = u(t) \overset{LT}{\longleftrightarrow} V_{in}(s) = 1/s$



From loop analysis:

$$0 = -V_{in}(s) + I(s)[R + Ls] - Li(0^-) \rightarrow I(s)[R + Ls] = V_{in}(s) + Li(0^-) \rightarrow$$

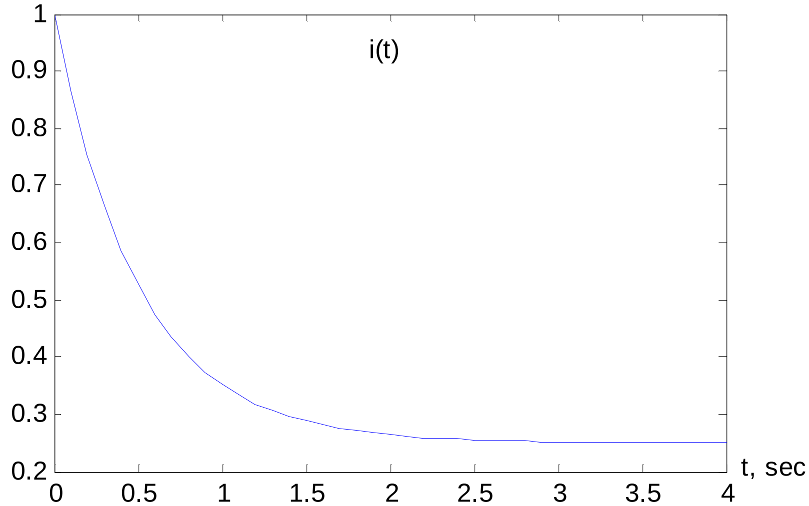
$$I(s) = \frac{V_{in}(s) + Li(0^-)}{R + Ls} = \frac{V_{in}(s)}{R + Ls} + \frac{Li(0^-)}{R + Ls} = \frac{(1/s)}{4 + 2s} + \frac{2(1)}{4 + 2s} = \frac{(1/s)}{4 + 2s} + \frac{2(1)}{4 + 2s}$$

$$= \frac{1}{2s(s+2)} + \frac{2}{2(s+2)} = \frac{0.5}{s(s+2)} + \frac{1}{s+2} = \frac{\alpha_1}{s} + \frac{\alpha_2}{s+2} + \frac{1}{s+2}$$

$$\alpha_1 = \left.\frac{0.5(s)}{s(s+2)}\right|_{s=0} = \left.\frac{0.5}{s+2}\right|_{s=0} = \frac{0.5}{0+2} = 0.25$$

$$\alpha_2 = \left.\frac{0.5(s+2)}{s(s+2)}\right|_{s=-2} = \left.\frac{0.5}{s}\right|_{s=-2} = \frac{0.5}{-2} = -0.25$$

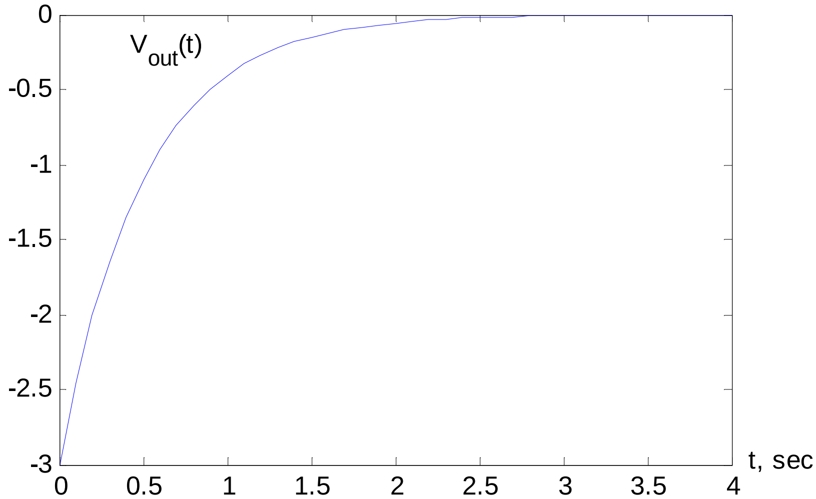
$$I(s)=\frac{0.25}{s}+\frac{-0.25}{s+2}+\frac{1}{s+2}=\frac{0.25}{s}+\frac{0.75}{s+2}\xleftrightarrow{LT^{-1}} i(t)=0.25u(t)+0.75e^{-2t}u(t)$$



B) $$V_{out}(t)=V_L(t)=L\frac{di(t)}{dt}\xleftrightarrow{LT} LsI(s)-Li(0^-)=[2s]I(s)-2=(2s)\left[\frac{0.25}{s}+\frac{0.75}{s+2}\right]-2$$

$$=\frac{0.5s}{s}+\frac{1.5s}{s+2}-2=0.5+\frac{1.5s}{s+2}-2=\frac{1.5s}{s+2}-1.5=\left[1.5-\frac{3}{s+2}\right]-1.5=-\frac{3}{s+2}$$

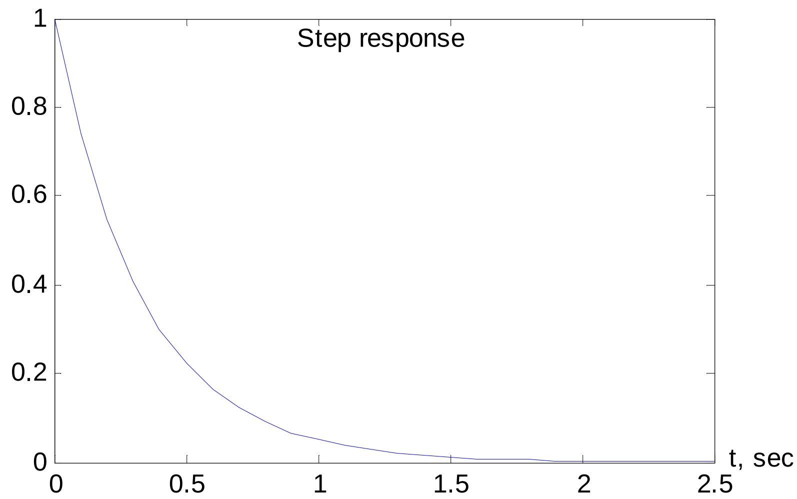
$$V_{out}(s)\xleftrightarrow{LT^{-1}} V_{out}(t)=-3e^{-2t}u(t)$$



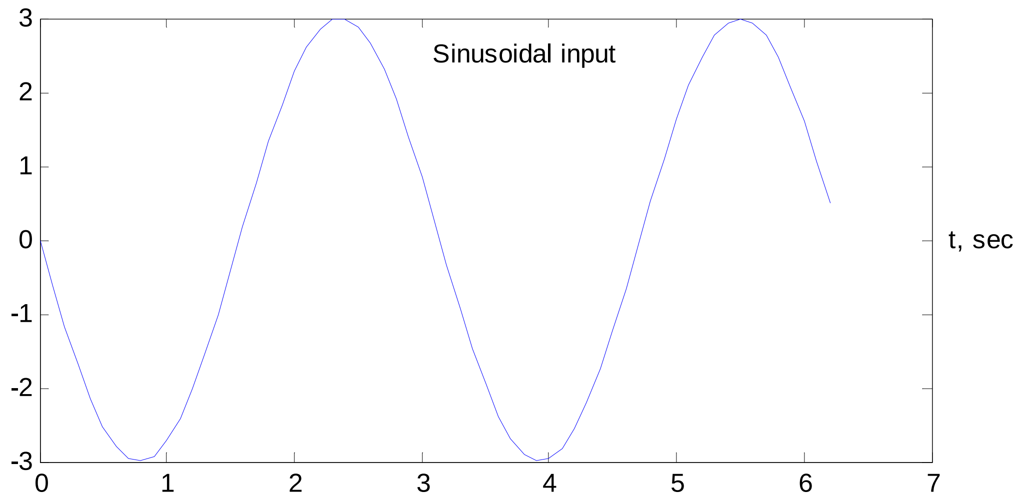
2)

A) $x(t) = u(t) \xleftrightarrow{LT} X(s) = 1/s$

$$Y(s) = X(s)H(s) = \frac{1}{s}\left(\frac{s}{s+3}\right) = \frac{1}{s+3} \xleftrightarrow{LT^{-1}} y(t) = e^{-3t}u(t)$$



B) $x(t) = 3\cos(2t + \pi/2)u(t)$



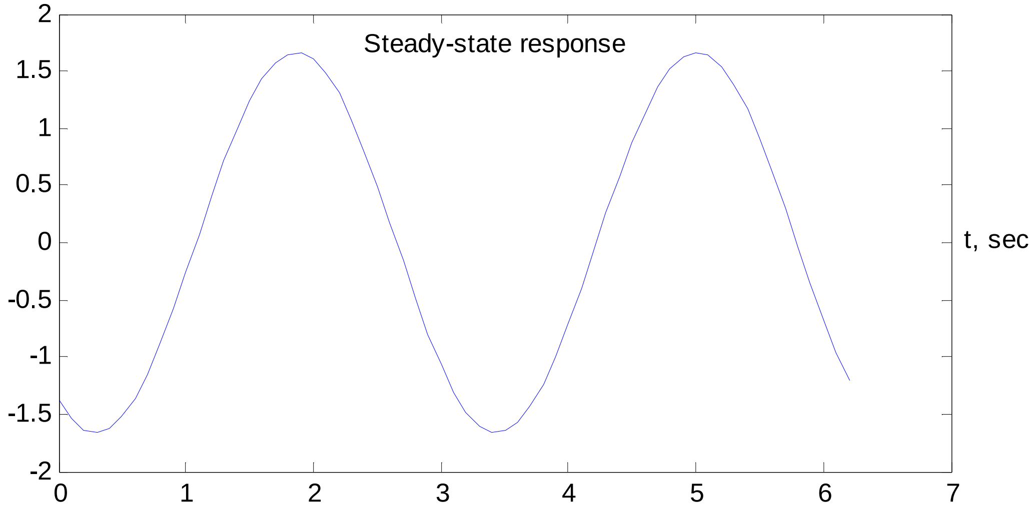
$$y_{ss}(t) = 3|H(j2)|\cos[2t + \pi/2 + \angle H(j2)]u(t)$$

$$H(j\omega) = H(s)\big|_{s=j\omega} = \frac{j\omega}{j\omega+3}$$

$$|H(j\omega)| = \frac{|j\omega|}{|j\omega+3|} = \frac{|\omega|}{\sqrt{3^2+\omega^2}} = \frac{|\omega|}{\sqrt{9+\omega^2}}$$

$$|H(j2)| = \frac{|2|}{\sqrt{9+2^2}} = \frac{2}{\sqrt{13}} = 0.5547$$

$$\angle H(j\omega) = \angle(j\omega) - \angle(j\omega + 3) = \pi/2 - \tan^{-1}(\omega/3) \quad \text{if } \omega > 0 \text{ rad/sec}$$
$$\angle H(j2) = \pi/2 - \tan^{-1}(2/3) = 0.9828 \text{ rad}$$
$$y_{ss}(t) = 3(0.5547)\cos[2t + \pi/2 + (0.9828)]u(t) = 1.6641\cos[2t + 2.5536]u(t)$$



3) Complete solution not available until lab assignment #4 is turned in.
   A) Assuming a second order low-pass filter with ω_c = 375π rad/sec, R = 285.8147 Ω and L = 0.1715 H.
   B) Assuming a second order band-pass filter with ω_0 = 500π rad/sec and BW = 250π rad/sec, R = 75.7881 Ω and L = 0.0965 H.