ECE 317
Summer 2005
Review Questions for Exam #3

General topics you should know for Exam #3:

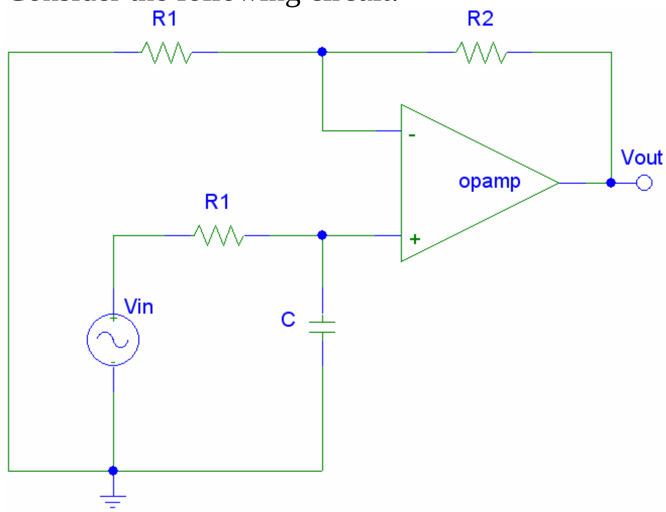
a) Passive filter design
   i) First order: low-pass, high-pass
   ii) Second order: low-pass, high-pass, band-pass, notch
   iii) Deriving the transfer function of a simple filter from its circuit
b) Active filter design
   i) First order: low-pass, high-pass, all-pass
   ii) Second order: low-pass, high-pass (Sallen & Key)
   iii) Deriving the transfer function of a simple filter from its circuit
c) Fourier analysis
   i) Magnitude response
   ii) Phase response
   i) Steady-state response to sinusoidal inputs
   ii) Basic understanding of Fourier series expansion
   iii) Parseval's theorem

<u>Questions:</u>

1) Design a first order filter with a constant magnitude response to produce a phase shift of $\pi/2$ radians at a frequency of 30 kHz using only 400 pF capacitors.
   A) Show how you designed the filter. Include the circuit schematic with all component values.
   B) Use PSpice to verify your design by plotting the magnitude response (in linear scale) and phase response (in degrees) of the filter.

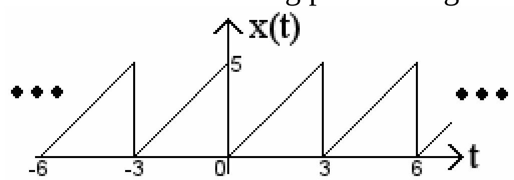
2) Design a third order active Butterworth high-pass filter with a cutoff frequency of 10 kHz using only 20 nF capacitors.
   <u>Hint:</u> Follow the same procedure as you would for a low-pass filter, but use high-pass circuits. The required Q value is 1.
   A) Show how you designed the filter. Include the circuit schematic with all component values.
   B) Use PSpice to verify your design by plotting the magnitude response (in dB) and phase response (in degrees) of the filter.

3) Consider the following circuit:



A) Show that the transfer function of the circuit matches that of a standard first order low-pass filter.
B) Express k_p and ω_c in terms of the circuit components.
C) Use this circuit to implement a low-pass filter with a cutoff frequency of 50 kHz and a DC gain of 1.5. Assume that the only available capacitor is 4.7 nF.

4) Consider the following periodic signal:



If the Fourier series coefficients of x(t) are:

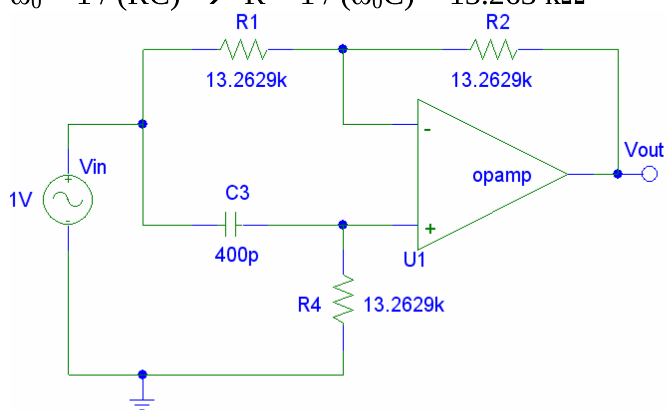
$$c_n = \begin{cases} c_0 & n = 0 \\ j\dfrac{5}{2\pi n} & n = \pm 1, \pm 2, \ldots \end{cases}$$

A) Calculate the value of c_0.
B) What property of the signal does c_0 represent?
C) What is the normalized average power contained in the DC component and the first 2 harmonics (i.e. up to n = 2)?
D) What percentage of the signal's total normalized average power is contained in the DC component and the first 2 harmonics?

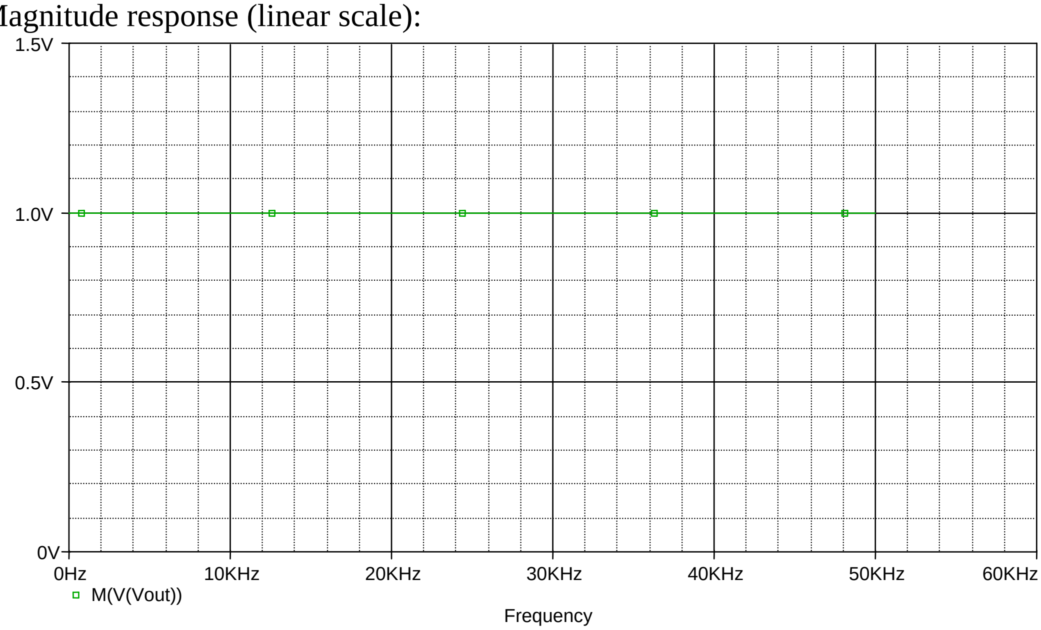
Solutions:

1)

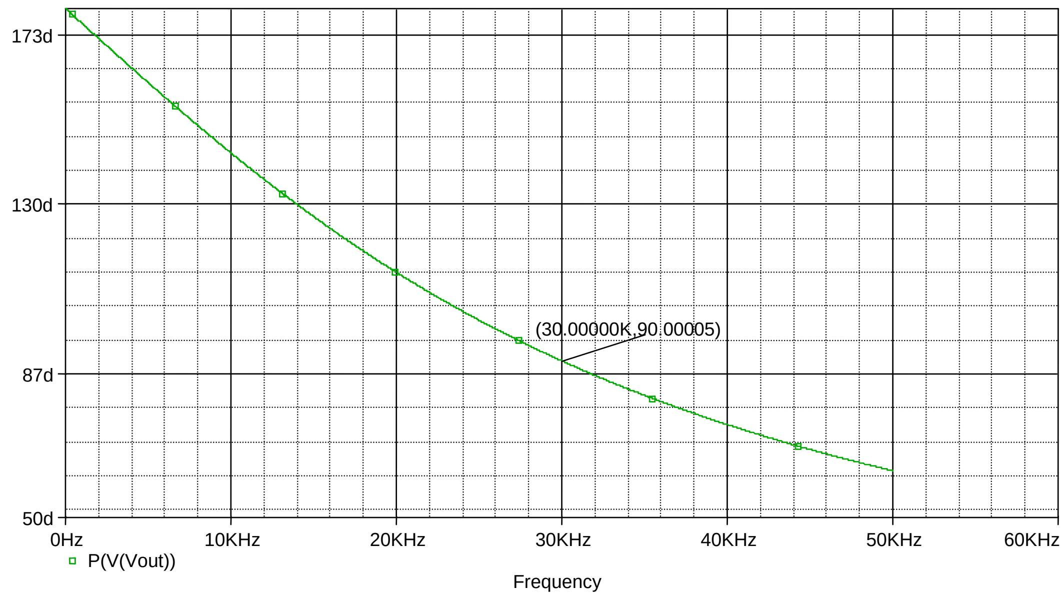
A) First order all-pass filter with $k_p > 0$ (non-inverting) and $f_0 = 30$ kHz
$C = 400$ pF
$k_p = 1$
$\omega_0 = 2\pi f_0 = 60{,}000\pi$ rad/sec
$\omega_0 = 1 / (RC) \rightarrow R = 1 / (\omega_0 C) = 13.263\ k\Omega$



B) Magnitude response (linear scale):



Phase response (degrees):



2)

A) C = 20 nF

$\omega_c = 2\pi f_c = 20{,}000\pi$ rad/sec

First order section:

$\omega_c = 1 / (RC) \rightarrow R = 1 / (\omega_c C) = 795.77\ \Omega$

Second order section:

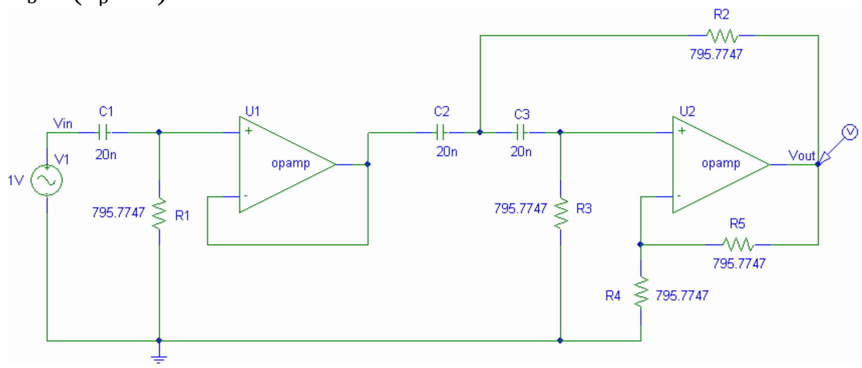
$Q = 1$

$\omega_0 = \omega_c$

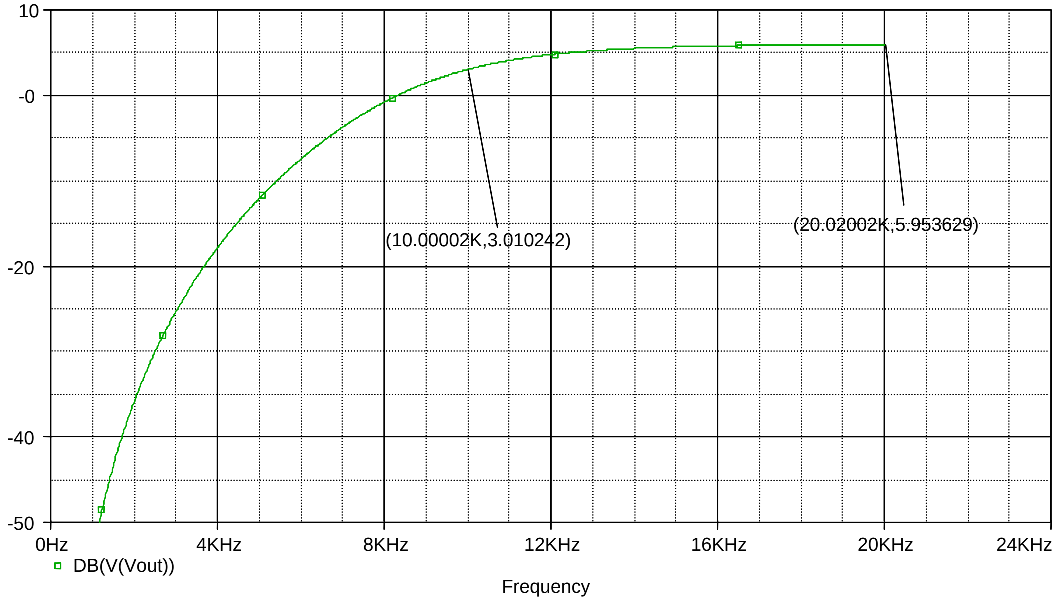
$\omega_c = 1 / (RC) \rightarrow R = 1 / (\omega_c C) = 795.77\ \Omega$

$k_p = 3 - (1/Q) = 2$

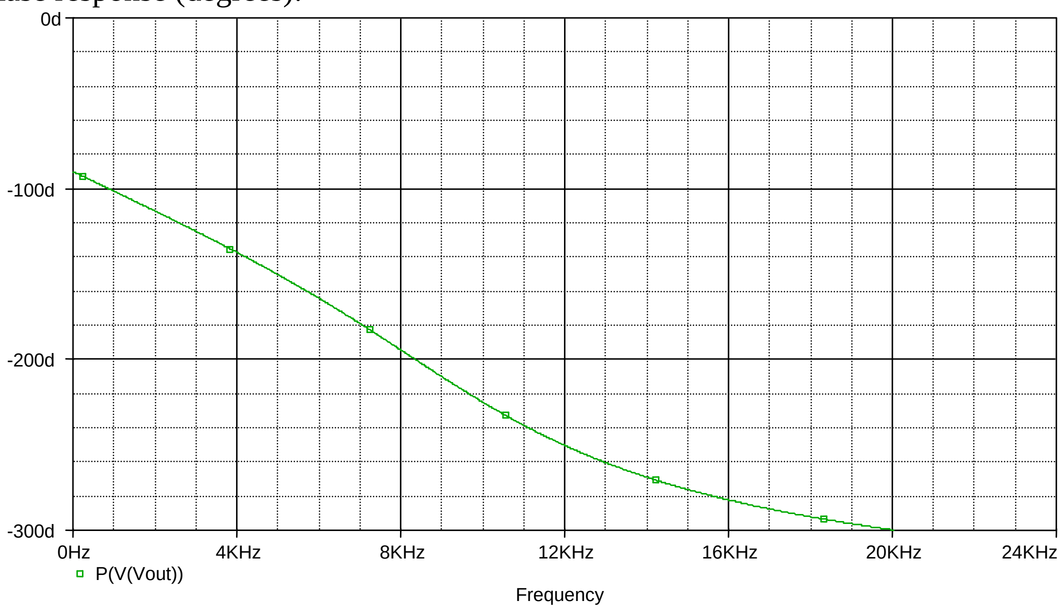
$R_b = (k_p - 1)R = 795.77\ \Omega$



B) Magnitude response (in dB):



Phase response (degrees):



3)

A) $$\frac{V_{out}}{V_{in}} = H(s) = \frac{\left(\frac{R_2}{R_1}+1\right)\left(\frac{1}{R_1 C}\right)}{s+\frac{1}{R_1 C}}$$

B) $k_p = \frac{R_2}{R_1}+1, \;\; \omega_c = \frac{1}{R_1 C}$

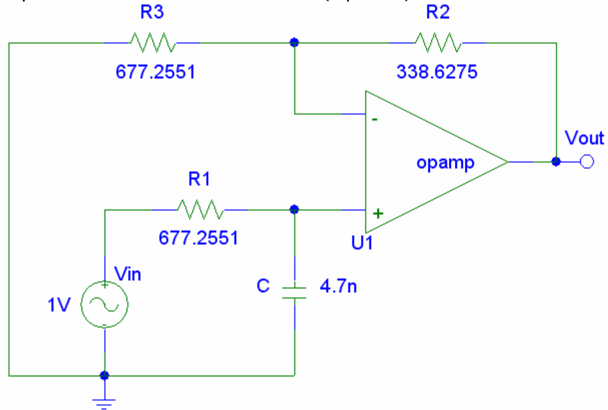
C) C = 4.7 nF

$\omega_c = 2\pi f_c = 100{,}000\pi$ rad/sec

$k_p = 1.5$

$\omega_c = 1 / (R_1 C)$ → $R_1 = 1 / (\omega_c C) = 677.26\ \Omega$

$k_p = R_2/R_1 + 1$ → $R_2 = (k_p - 1)R_1 = 338.63\ \Omega$



4)

A) $T_0 = 3$ seconds

For $0 \le t \le T_0$, the slope = 5/3 and the y-intercept = 0.

$$c_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\,dt = \frac{1}{3}\int_0^3 \frac{5}{3}t\,dt = 2.5$$

B) c_0 is the average value of the signal or the DC component of the Fourier series expansion.

C) $$P = \sum_{n=-\infty}^{\infty}|c_n|^2 = c_0^2 + 2\sum_{n=1}^{\infty}|c_n|^2 = (2.5)^2 + 2\sum_{n=1}^{\infty}\left|j\frac{5}{2\pi n}\right|^2$$

For DC and the first two harmonics:

$$P_2 = (2.5)^2 + 2\sum_{n=1}^{2}\left|j\frac{5}{2\pi n}\right|^2 = 6.25 + 2\left(\left|j\frac{5}{2\pi(1)}\right|^2 + \left|j\frac{5}{2\pi(2)}\right|^2\right)$$

$P_2 = 7.8331$ W

D) $$P = \frac{1}{T_0}\int_{t_0}^{t_0+T_0}|x(t)|^2\,dt = \frac{1}{3}\int_0^3\left|\frac{5}{3}t\right|^2 dt = \frac{1}{3}\int_0^3\frac{25}{9}t^2\,dt = \frac{25}{27}\int_0^3 t^2\,dt = \frac{25}{27}\frac{t^3}{3}\bigg|_0^3 = \frac{25}{81}\left(3^3 - 0^3\right)$$

$$P = \frac{25}{81}(27) = \frac{25}{3} = 8.3333\ \text{W}$$

$$\frac{7.8331}{8.3333}(100\%) = 93.9977\ \%\ \text{of the total normalized average power}$$