Limit to Radiative Decay of the Hydrogen Atom of Classical Electrodynamics

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An analysis of forces acting in a classical electrodynamics model of a bound hydrogen atom is presented, including those due to orbital motion of intrinsic magnetic moments, and accounting for propagation delay of the forces between the constituent particles. Orbital motion of the electron intrinsic magnetic moment is shown to result in a radial force on the proton that is not directly balanced by a similar-magnitude force on the electron. Propagation delay of the force results in a non-radial aberrational component capable of negating the radiation reaction force expected from energy and momentum conservation principles. Radiative decay of the classical atom need thus not occur. It is shown that the electron-proton separation at which the aberrational and radiation reactive forces cancel is on the order of the Bohr ground state atomic radius. Finally, it is argued that this unbalanced force will give rise to oscillatory motions that may plausibly account for the apparent wave nature of the electron.

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I. INTRODUCTION AND OUTLINE

In this work forces acting on the constituent particles of hydrogen, an electron and a proton, are derived from the classical theory of electromagnetism. It is assumed that the hydrogen atom sole constituents are an electron and a proton that are point-like at the scale of the analysis, have well-defined positions, and have mass and electric charge of accepted values. Also, of equal importance, they each possess intrinsic angular momentum and associated magnetic moment of accepted and experimentally measured values.

A brief overview of the classical model of the hydrogen atom, excluding intrinsic spins and based solely on the Coulomb attraction between the electron and proton, is first provided. The relationships between the constituent particle positions and velocities is defined for the case of circular orbits. Then, additional electromagnetic forces as are derivable from the motion of the constituent particle intrinsic magnetic moments are derived and compared in magnitude and direction with the Coulomb forces, for the special case of circular orbits. It is shown that the orbital motion of the electron intrinsic magnetic moment causes a radial force on the proton that is not balanced by an equal-magnitude force on the electron due to the orbital motion of the proton intrinsic magnetic moment. Possible implications of this unbalanced force are discussed.

Then, the effect of propagation delay on the force due to the orbital motion of the electron intrinsic magnetic moment is assessed. It is shown that the force on the proton has a resulting aberrational component at typical classical atomic radii that is comparable to the classically-expected radiation reaction force on the electron, and how a stable orbit might result.

II. OVERVIEW OF THE CLASSICAL HYDROGEN ATOM

The magnitude of the Coulomb force, F, acting between two charged particles of equal charge magnitude, e, and separated by a distance R, is, in Gaussian units,

$$F = \frac{e^2}{R^2} \tag{1}$$

For particle charges of opposite polarity the Coulomb force is attractive and, neglecting effects due to propagation delay, is oppositely directed for each particle, so we can write

$$\boldsymbol{F}_e = -\boldsymbol{F}_p \tag{2}$$

where $\boldsymbol{F}_e$ and $\boldsymbol{F}_p$ are the vector Coulomb forces on the electron and proton.

It is well known that a force law such as that given by (2) results in conservative motion, and for the case of inverse-square-law force such as given by (1), known as Kepler's problem, results in regular elliptical orbits. In this work we confine our attention to the case of circular orbits, however.

We suppose the electron and proton are in a circular orbit around their common center of mass. Then balancing the centrifugal acceleration of the electron with the Coulomb attraction from the proton yields

$$m_e \frac{{v_e}^2}{R_e} = \frac{e^2}{R^2} \tag{3}$$

where m_e and v_e are the electron mass and velocity measured in the center-of-mass frame, R_e is the electron distance from the center of mass and R is the electron-proton separation. We also have that

$$R = R_e + R_p \tag{4}$$

where R_p is the proton distance from the center of mass, and from the definition of the center of mass

$$m_e R_e = m_p R_p \tag{5}$$

so

$$R = \left(1 + \frac{m_e}{m_p}\right) R_e = \left(\frac{m_p + m_e}{m_p}\right) R_e \tag{6}$$

or

$$R_e = \left(\frac{m_p}{m_p + m_e}\right) R \tag{7}$$

Also, in order for the electron and proton to remain opposite each other across the center of mass, we must have that

$$\frac{\boldsymbol{v}_e}{R_e} = -\frac{\boldsymbol{v}_p}{R_p} \tag{8}$$

so

$$\boldsymbol{v}_p = -\frac{m_e}{m_p}\boldsymbol{v}_e \tag{9}$$

If $\boldsymbol{v}$ is the electron velocity relative to the proton, then

$$\boldsymbol{v} = \boldsymbol{v}_e - \boldsymbol{v}_p \tag{10}$$

or

$$\boldsymbol{v} = \boldsymbol{v}_e + \frac{m_e}{m_p}\boldsymbol{v}_e = \left(\frac{m_p + m_e}{m_p}\right)\boldsymbol{v}_e \tag{11}$$

Substituting (7) into (3):

$$m_e \frac{m_p + m_e}{m_p}\frac{{v_e}^2}{R} = \frac{e^2}{R^2} \tag{12}$$

and with (11)

$$m_e \frac{m_p + m_e}{m_p}\left(\frac{m_p}{m_p + m_e}\right)^2 \frac{v^2}{R} = \frac{e^2}{R^2} \tag{13}$$

And so, in terms of the electron-proton separation,

$$v^2 = \frac{e^2}{m_r R} \tag{14}$$

where

$$m_r = \frac{m_e m_p}{m_p + m_e} \tag{15}$$

is the "reduced" mass of the electron. Alternatively, from (7), we also have

$$v^2 = \frac{e^2}{R_e}\left(\frac{m_e m_p}{m_p + m_e}\right)^{-1}\left(\frac{m_p}{m_p + m_e}\right) \tag{16}$$

so, in terms of the electron orbital radius in the center-of-mass frame,

$$v = \frac{e}{\sqrt{m_e R_e}} \tag{17}$$

Using (11) again, the electron velocity as measured in an inertial reference frame with origin at the center of mass between the particles is

$$v_e = \frac{m_p}{m_p + m_e}\frac{e}{\sqrt{m_e R_e}} \tag{18}$$

For our work it will be instructive to consider the relationship between forces having differing dependencies on the electron-proton separation, and therefore some standard will be needed as representative of atomic separations. For this we will use the ground-state radius from the Bohr atomic theory for hydrogen given by

$$R_B = \frac{\hbar^2}{m_e e^2} \tag{19}$$

where $\hbar = h/2\pi$ is the reduced Planck's constant.

We will not use the Bohr model otherwise.

Because an electron in a circular orbit will radiate electromagnetic energy and momentum, conservation considerations argue that a compensating force known as the radiation reaction force will cause the electron to decelerate. The radiation reaction is given by [1]

$$\boldsymbol{F}_{\text{rad}} = \frac{2}{3}\frac{e^2}{c^3}\ddot{\boldsymbol{v}} \tag{20}$$

For the circular orbit, for the electron, we obtain for the radiation rsesistance force magnitude

$$\begin{aligned} F_{\text{rad}} &= |\boldsymbol{F}_{\text{rad}}| = \frac{2}{3}\frac{e^2}{c^3}\frac{{v_e}^3}{{R_e}^2} \\ &= \frac{2}{3}\frac{e^2}{c^3}\frac{1}{{R_e}^2}\left(\frac{m_p}{m_p + m_e}\cdot\frac{e}{\sqrt{m_e R_e}}\right)^3 \end{aligned} \tag{21}$$

or

$$F_{\rm rad} = \frac{2}{3}\frac{e^5}{c^3}{R_e}^{-7/2}\left(\frac{m_p}{m_p+m_e}\cdot\frac{1}{\sqrt{m_e}}\right)^3 \quad (22)$$

or

$$F_{\rm rad} = \frac{2}{3}\frac{e^5}{c^3}R^{-7/2}\left(\frac{m_p}{m_p+m_e}\right)^{-7/2}\left(\frac{m_p}{m_p+m_e}\cdot\frac{1}{\sqrt{m_e}}\right)^3 \quad (23)$$

or

$$F_{\rm rad} = \frac{2}{3}\frac{e^5}{c^3}R^{-7/2}\left(\frac{m_p}{m_p+m_e}\right)^{-1/2}\left(\frac{1}{\sqrt{m_e}}\right)^3 \approx \frac{2}{3}\frac{e^5}{c^3}R^{-7/2}m_e^{-3/2} \quad (24)$$

III. ELECTRIC FORCES DUE TO ORBITAL MOTION OF INTRINSIC MAGNETIC MOMENTS

When the classical atomic model as outlined above was originally considered, it was not yet known that the electron and proton possess in addition to their electrical charge, intrinsic magnetic moments and associated angular momenta, or "spin". It is also known now, as shown in the quantum mechanical analysis of the hydrogen atom based on the Shrödinger equation, that the particle spins and intrinsic magnetic moments are important in determining the allowed states. In particular the stronger electron intrinsic magnetic moment plays an important role in the Shrödinger picture, in interacting with its own magnetic moment due to its orbital motion around the proton. This "spin-orbit" interaction of the electron intrinsic and orbital magnetic moments is the second most energetically important interaction after Coulomb attraction. Therefore, in light of this not-so-recent discovery of the presence of important interactions other than Coulomb attraction, and such as might be anticipated to be amenable to a classical lectrodynamics analysis, it would seem reasonable to revisit the classical atomic model to assess what effects the intrinsic moments might have.

In the present work we will focus primarily on electrical forces that result due to the motion of the electron intrinsic magnetic moment. Electrical forces are inherently more interesting than magnetic forces because unlike magnetic forces, which can do no work because they act perpendicularly to the velocity, electrical forces have the capacity to do work. Electrical forces derived from the electron magnetic moment are anticipated to be more important than similar forces due to the proton, furthermore, due to the much larger intrinsic magnetic moment of the electron compared to that of the proton.

Our treatment will use a classical representation for intrinsic angular momentum that ignores spatial quantization. The intrinsic angular momentum will be represented simply as

$$\boldsymbol{s} = \frac{\hbar}{2}\hat{\boldsymbol{s}} \quad (25)$$

where $\hat{\boldsymbol{s}}$ is a unit-magnitude orientation vector. The intrinsic magnetic moments are then of the form

$$\boldsymbol{\mu} = \frac{ge}{2mc}\boldsymbol{s} = \frac{ge}{2mc}\frac{\hbar}{2}\hat{\boldsymbol{s}} \quad (26)$$

where the quantity g is the "g factor", which has a fixed value depending on the particle type. For the electron

$$g \approx 2.0024 \approx 2 \quad (27)$$

In the presence of moving particles possessing magnetic dipole moments with possibly time-varying orientations, an electric field will generally arise, as will now be shown.

The vector potential $\boldsymbol{A}$ due to a magnetic moment $\boldsymbol{m}$ at a field point outside the source region, separated from $\boldsymbol{m}$ by $\boldsymbol{r}$ is [1]

$$\boldsymbol{A} = \frac{\boldsymbol{m}\times\boldsymbol{r}}{R^3} \quad (28)$$

and from the vector and scalar potentials the electric field is given by

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t} - \nabla\Phi \quad (29)$$

For an elemental magnetic dipole the scalar potential vanishes, so the electric field due to the magnetic moment is simply

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t} \quad (30)$$

The electric field due to a moving magnetic moment is thus

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{m}\times\boldsymbol{r}}{R^3}\right) \quad (31)$$

or

$$\boldsymbol{E} = \frac{3}{cR^4}\left(\boldsymbol{m}\times\boldsymbol{r}\right)\frac{\partial R}{\partial t} - \frac{1}{cR^3}\frac{\partial}{\partial t}\left(\boldsymbol{m}\times\boldsymbol{r}\right) \quad (32)$$

We consider first the case of circular orbits. For the electron in a circular orbit around the proton, $\frac{\partial R}{\partial t} \equiv 0$ and (32) becomes

$$\boldsymbol{E} = -\frac{1}{cR^3}\frac{\partial}{\partial t}(\boldsymbol{m}\times\boldsymbol{r}) \quad (33)$$

or

$$\boldsymbol{E} = -\frac{1}{cR^3}\left(\frac{\partial \boldsymbol{m}}{\partial t}\times\boldsymbol{r} - \frac{\partial \boldsymbol{r}}{\partial t}\times\boldsymbol{m}\right) \quad (34)$$

The first term will be nonzero for a precessing spin moment while the second will be nonzero due to the orbital motion of the electron around the proton. The second term will be the larger due to the fact that the orbital frequency greatly exceeds the magnetic moment precessional frequency. We will therefore investigate the second term prior to the first. The electric field component due to the motion of the electron magnetic moment relative to the proton, at the proton, may be rewritten as

$$\boldsymbol{E}_{\mu\,\mathrm{vel}} = \frac{1}{cR^3}(\boldsymbol{v}\times\boldsymbol{\mu}_e) \quad (35)$$

For a proton also orbiting about the common center of mass with the electron, we can calculate the work done on the proton by this field as

$$W_{\mu\,\mathrm{vel}} = \int \boldsymbol{F}\cdot d\boldsymbol{r}_p = \int \boldsymbol{F}\cdot\boldsymbol{v}_p dt = \int e\boldsymbol{E}_{\mu\,\mathrm{vel}}\cdot\boldsymbol{v}_p dt \quad (36)$$

or

$$W_{\mu\,\mathrm{vel}} = \frac{e}{cR^3}\int ((\boldsymbol{v}_e - \boldsymbol{v}_p)\times\boldsymbol{\mu}_e)\cdot\boldsymbol{v}_p dt \quad (37)$$

In the scalar triple product we can rearrange the dot and cross products as

$$W_{\mu\,\mathrm{vel}} = \frac{e}{cR^3}\int (\boldsymbol{v}_p\times(\boldsymbol{v}_e - \boldsymbol{v}_p))\cdot\boldsymbol{\mu}_e dt \quad (38)$$

and, since $\boldsymbol{v}_p = -\frac{m_e}{m_p}\boldsymbol{v}_e$ (neglecting propagation delay), we have that the integrand is identically zero and so

$$W_{\mu\,\mathrm{vel}} = 0 \quad (39)$$

Similarly, the electric field at the proton due to the first term of (34) is

$$\boldsymbol{E}_{\dot{\mu}} = \frac{1}{cR^3}\left(\frac{\partial \boldsymbol{\mu}_e}{\partial t}\times\boldsymbol{r}\right) \quad (40)$$

and with

$$\frac{\partial \boldsymbol{\mu}_e}{\partial t} = \frac{d\boldsymbol{\mu}_e}{dt} \equiv \dot{\boldsymbol{\mu}}_e \quad (41)$$

yields

$$W_{\dot{\mu}} = \frac{e}{cR^3}\int (\dot{\boldsymbol{\mu}}_e\times\boldsymbol{r})\cdot\boldsymbol{v}_p dt \quad (42)$$

or

$$W_{\dot{\mu}} = \frac{e}{cR^3}\int \dot{\boldsymbol{\mu}}_e\cdot(\boldsymbol{r}\times\boldsymbol{v}_p)\, dt \quad (43)$$

For circular or elliptical orbit the radius and velocity vectors are in the plane of the orbit, so their cross product points perpendicularly to the orbital plane. For precessional motion of the magnetic moment in a magnetic field due to the orbital motion of the charge, $\dot{\boldsymbol{\mu}}_e$ will also be in the plane of the orbit and so the integrand in (38) will be zero. However, $\dot{\boldsymbol{\mu}}_e$ may lie out of the orbital plane if $\boldsymbol{\mu}_e$ is experiencing a torque due to the proton intrinsic magnetic moment and orientation. Therefore, for circular or elliptical orbits, the electric field due to an orbiting magnetic moment does no work, but an electric field due to a changing orientation of the moment may do work.

Next we consider non-circular orbits, where the first term in (32) may be nonzero. The electric field at the proton location due to this term is

$$\boldsymbol{E}_{\dot{R}} = \frac{3}{c}\frac{1}{R^4}\frac{\partial R}{\partial t}(\boldsymbol{\mu}_e\times\boldsymbol{r}) \quad (44)$$

The work done on the proton due to this term is

$$W_{\dot{R}} = \frac{3e}{c}\int \frac{1}{R^4}\frac{\partial R}{\partial t}(\boldsymbol{\mu}_e\times\boldsymbol{r})\cdot\boldsymbol{v}_p dt \quad (45)$$

which may also be written as

$$W_{\dot{R}} = \frac{3e}{c}\int \frac{1}{R^4}\frac{\partial R}{\partial t}\boldsymbol{\mu}_e\cdot(\boldsymbol{r}\times\boldsymbol{v}_p)\, dt \quad (46)$$

From (44) it is clear that $\boldsymbol{E}_{\dot{R}}$ has no radial component. Work may be done by $\boldsymbol{E}_{\dot{R}}$ perpendicular to the radial direction, however, and depending on the orientation of $\boldsymbol{\mu}_e$, either in the orbital plane or perpendicularly to it. So, radial motion driven by an unbalanced radial force on the proton such as that of (35) will result in energy being subtracted or added to the proton orbital motion and may drive an out-of-plane proton motion as well, depending on the orientation of the electron spin.

It is to be noted that in a bound system such as under consideration here, the presence of seemingly unopposed forces on the proton compared to the electron does not result in energy and momentum non-conservation in a long-term sense. If the unopposed force on the proton results in a change in position and velocity of the proton, energy and momentum conserving forces will be transmitted back to the electron via the Coulomb binding force. Thus, the electron spin will interact classically with its own orbit via mediating forces on the proton and their concomitant effects back on the electron.

IV. EFFECT OF PROPAGATION DELAY ON ELECTRIC FORCE DUE TO ORBITAL MOTION OF MAGNETIC MOMENT

In the previous section we determined that the differential work done on either particle by the orbital motion of the intrinsic magnetic moment of the other (i.e., the integrand in (38)) is identically zero for the case of electron velocity being exactly antiparallel to the proton velocity. However the proton velocity in question is properly associated with the laboratory present time, while the electron velocity is properly the so-called retarded velocity associated with the time one light-propagation delay in the past. It is worthwhile to ask therefore whether the resultant deviation of the electron velocity from being exactly antiparallel to the proton velocity will give rise to a significant "aberrational" force.

For the case of non-relativistic circular orbital motion, the propagation delay, D, is approximately that associated with the orbital radius as

$$D = \frac{R}{c} \tag{47}$$

The angular change of the velocity vector during the propagation delay is

$$\phi_D = \frac{vD}{R} = \frac{v}{c} \tag{48}$$

A. Aberrational Force on Proton due to Orbital Motion of Electron Intrinsic Magnetic Moment

To get an idea of the possible magnitude of the force resulting from non-instantaneity, we will compute the force magnitude based on (38) for the case of a circular orbit. The force acting on the proton due to the orbital motion of the electron intrinsic magnetic moment is given by

$$\boldsymbol{F}_{\mu\,\mathrm{vel}} = e\boldsymbol{E}_{\mu\,\mathrm{vel}} = \frac{e}{cR^3}\left(\boldsymbol{v} \times \boldsymbol{\mu}_e\right) \tag{49}$$

For electron spin perpendicular to the orbital plane the force magnitude is

$$F_{\mu\,\mathrm{vel}} = \frac{e}{cR^3}\left(v\mu_e\right) \approx \frac{e}{cR^3}\left(\frac{e}{\sqrt{m_e R}}\frac{e\hbar}{2m_e c}\right) = \frac{e^3\hbar}{2c^2}R^{-7/2}m_e^{-3/2} \tag{50}$$

where the approximation is to neglect the distinction between the electron actual and reduced mass. Accounting for propagation delay, the component along the direction of the proton motion is approximately

$$F_{\|} \approx \sin\left(\frac{v}{c}\right)\frac{e^3\hbar}{2c^2}R^{-7/2}m_e^{-3/2} \approx \frac{v}{c}\frac{e^3\hbar}{2c^2}R^{-7/2}m_e^{-3/2} \tag{51}$$

or

$$F_{\|} \approx \frac{1}{c}\frac{e}{\sqrt{m_e R}}\frac{e^3\hbar}{2c^2}R^{-7/2}m_e^{-3/2} = \frac{e^4\hbar}{2c^3}R^{-4}m_e^{-2} \tag{52}$$

For comparison, we consider the relative magnitude of the force due to radiation reaction on the electron. By equating the aberrational force and the radiation reaction force, which will be seen to have differing dependencies on the electron-proton separation, we can determine the separation value at which the two forces will be of equal magnitude.

Equating the radiation reaction on the electron, as given by (24), with the force due to the moving electron intrinsic magnetic moment, that is, evaluating

$$F_{\mathrm{rad}} = F_{\|} \tag{53}$$

yields

$$\frac{2}{3}\frac{e^5}{c^3}R^{-7/2}{m_e}^{-3/2} \approx \frac{e^4\hbar}{2c^3}R^{-4}m_e^{-2} \tag{54}$$

So

$$R^{1/2}m_e^{1/2} \approx \frac{3}{4}\frac{c^3}{e^5}\frac{e^4\hbar}{c^3} \tag{55}$$

or

$$R^{1/2}{m_e}^{1/2} \approx \frac{3}{4}\frac{\hbar}{e} \tag{56}$$

or

$$R \approx \frac{9}{16}\frac{\hbar^2}{m_e e^2} \tag{57}$$

is the electron-proton separation where the force on the proton due to the electron intrinsic magnetic moment orbital motion is similar in magnitude to the radiation reaction force on the electron. For further comparison, (57) can be expressed in terms of the Bohr radius as

$$R \approx \frac{9}{16}R_B \tag{58}$$

So, for the electron spin model and orientation under analysis, the range where the aberrational force on the proton is equal to the radiation reaction force on the electron is approximately one-half the Bohr radius. Also, since the aberrational force increases more rapidly with decreasing range than the radiation reaction force, the aberrational force will be the larger for all smaller radii than (58).

B. Effect of Aberrational Force on Proton due to Orbital Motion of Electron Intrinsic Magnetic Moment

Previously it was shown that the magnitude of the aberrational force on the proton, due to the propagation-delayed force due to the motion of the electron intrinsic magnetic moment, was of the same order of magnitude as the radiation reaction force on the electron, for orbital radii on the order of the Bohr ground state radius. On the other hand, the respective similar aberrational force on the electron due to the relative motion of the proton intrinsic magnetic moment will be much less, given the much smaller intrinsic magnetic moment of the proton. Of the two aberrational forces, then, the one on the proton due to the electron is clearly the more interesting and worthy of further investigation.

Now, it is usually considered fatal to the classical model of the hydrogen atom that the unopposed force of radiation reaction on the electron will cause radiative decay in a small fraction of a second. At first glance our aberrational force, acting on the proton rather than the electron, would not seem to remedy that situation. However conservative motion will generally occur if the forces on the two particles have no non-central component and are of equal magnitudes, as well as attractive. That is, as in (2), conservative motion will occur if

$$\boldsymbol{F}_p = -\boldsymbol{F}_e \tag{59}$$

For illustration of the applicability of this principle to the problem at hand, suppose that the only forces acting are the opposing Coulomb forces and the radiation reaction force on the electron and aberrational force on the proton. Then, if we define $\boldsymbol{F}_{\text{Coul}}$ as the Coulomb force acting on the proton, (59) becomes

$$\boldsymbol{F}_{\text{Coul}} + \boldsymbol{F}_{\text{ab}} = -(-\boldsymbol{F}_{\text{Coul}} + \boldsymbol{F}_{\text{rad}}) \tag{60}$$

So, conservative motion will occur if

$$\boldsymbol{F}_{\text{ab}} = -\boldsymbol{F}_{\text{rad}} \tag{61}$$

where $\boldsymbol{F}_{\text{ab}}$ is the aberrational force on the proton and $\boldsymbol{F}_{\text{rad}}$ is the radiation reaction force on the electron. The aberrational force on the proton has effectively canceled the radiation reaction force on the electron.

Above, the radiation reaction force on the proton and aberrational force on the electron were neglected. These forces are very small compared to those identified explicitly in (60). However, the also-neglected radial component of the electrical force on the proton due to the orbital motion of the electron magnetic moment is much larger than the aberrational or radiation reaction forces, in that the analyzed aberrational force is merely the small non-radial component of the entirety of the force due to the moving electron intrinsic moment, and so the larger radial component should not be ignored. It is at least less than obvious then what force if any might cancel the latter force. One possibility might be an aberrational component of the Coulomb force. Given that the Coulomb force is much larger than the force due to the orbital motion of intrinsic moments, it might be expected that an aberrational component of the Coulomb force could be sufficiently large. But, evaluation of this force shows that it is very small; smaller than the radiation reaction force on the electron at the Bohr radius. This is so because properly time-delayed potentials and forces moving of point charges (i.e., the Liénard-Wiechart potentials and forces) include a term for the linear motion of the source relative to the field point, which corrects the lag to the first order [3]. It is an interesting feature of the electric force due to motion of magnetic moment that the correction for linear motion present in the properly-lagged Coulomb force integrates out of its expression [2].

Also, calculation of the force on the electron due to the magnetic field due to the orbiting proton shows it cannot be equated to the force on the proton due to the electron intrinsic magnetic moment orbital motion. The prior force is less than one one-hundred-thousandth as strong as the latter force. The force on the electron due to the orbital motion of the proton intrinsic magnetic moment is much smaller still due to both the smaller intrinsic magnetic moment of the proton compared to the electron as well as the smaller velocity of the proton. So, from these considerations we conclude that the exact equality of the electron and proton force magnitudes per (59) cannot hold and so exact circular conservative orbital motion cannot occur.

Although we do not have the exact opposition of forces as required by (59), the deviation from equality in actuality is quite small. The ratio of $F_{\mu\,\text{vel}}$ to the Coulomb attractive force may be evaluated as

$$\frac{F_{\mu\,\text{vel}}}{F_{\text{Coul}}} \approx \left(\frac{e^3\hbar}{2c^2} R^{-7/2} {m_e}^{-3/2}\right)\left(\frac{e^2}{R^2}\right)^{-1} \tag{62}$$

or

$$\frac{F_{\mu\,\text{vel}}}{F_{\text{Coul}}} \approx \frac{e\hbar}{2c^2} R^{-3/2} {m_e}^{-3/2} \tag{63}$$

At the Bohr ground state radius this becomes

$$\frac{F_{\mu\,\text{vel}}}{F_{\text{Coul}}} \approx \frac{e\hbar}{2c^2} {m_e}^{-3/2} \left(\frac{\hbar^2}{m_e e^2}\right)^{-3/2} \tag{64}$$

or

$$\frac{F_{\mu\,\text{vel}}}{F_{\text{Coul}}} \approx \frac{e^4}{c^2\hbar^2} \tag{65}$$

or

$$\frac{F_{\mu\,\mathrm{vel}}}{F_{\mathrm{Coul}}} \approx 5\times 10^{-5} \qquad (66)$$

So, it would seem reasonable to treat the motion as a small perturbation from uniform circular motion. Since the additional force due to the orbiting electron intrinsic magnetic moment is radially directed, except for the small aberrational residual, it will cause a change in the electron-proton separation with time. However we also had previously that a change in the electron-proton separation gives rise via the electron intrinsic magnetic moment to an additional electric field component given by (32) as

$$\boldsymbol{E}_{\dot{R}} = \frac{3}{cR^4}\left(\boldsymbol{\mu}_e \times \boldsymbol{r}\right)\frac{\partial R}{\partial t} \qquad (67)$$

This force is never radially directed, so there will be no direct effect on the electron-proton separation due to it, but it will affect the separation indirectly. It will add or subtract energy and momentum to the proton orbit in accordance with the velocity of the electron-proton separation. This will in turn affect both the energy and angular momentum of the electron and hence the orbital separation and circularity. Because it depends on a velocity however it does not act directly in response to the radial force but rather to a time integral of it. A finite force will not occur until the proton velocity is finite which also implies a finite displacement of the proton from its circular orbit with constant angular momentum. The finite displacement of the proton will cause a deviation in the Coulomb force felt by the electron. So, we have that energy will exchange dynamically between the potential energy of the Coulomb attraction and the kinetic energy of the electron. Analysis of the dynamics of this motion would seem an interesting task, but is beyond the scope of the current work. However, we do note that the force due to the motion of the intrinsic magnetic moment involves Planck's constant through the value of the intrinsic spin, and so any possible stable orbits under the action of these forces are likely to involve Planck's constant as well.

C. Notes on the Proper Form of the Vector Potential of a Moving Magnetic Dipole

In our definition of the vector potential for a magnetic moment, as given by (28), we cited Jackson [1], but the derivation there is from a magnetostatics point of view and makes no attempt to include propagation delay effects. However, in [2], a derivation which results in a formally identical result as in (28) is provided, starting from the Liénard-Wiechert potentials and so directly including propagation delay, and therefore proving that the proper incorporation of propagation delay is to evaluate the source position and velocity as being at the retarded time. Therefore, our use of (28) here is entirely within the scope of its applicability.

V. CONCLUSION

It has been shown that electric forces due to the orbital motion of the electron intrinsic magnetic moment, in a classical atomic model, can prevent radiative decay of the atom as well as introduce deviations from exact Keplerian motion. The latter may plausibly provide a classical basis for stable orbital motions with only discrete energy and angular momentum.

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