

## Introductory Algebra

What you should know

MyMathLab

course name: Algebra 2008-2009

course id : butterworth62075

Elvira Leonore Heybey

## 1.1: Tips for Success in Mathematics

Do your homework

Learn from your mistakes and be patient with yourself

Know how to get help if you need it  
ARC

Read your textbook before class

Don't be afraid to ask questions

## 1.2: Symbols and Sets of Numbers

### Natural Numbers

The set of **natural numbers** is  $\{1, 2, 3, 4, 5, 6, \dots\}$ .

### Whole Numbers

The set of **whole numbers** is  $\{0, 1, 2, 3, 4, \dots\}$ .

### Integers

The set of **integers** is  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

### Rational Numbers

$$\left\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers and } \underline{\underline{b \neq 0}} \right\}$$

$$\frac{a}{b}$$

$$\frac{6}{2} = 3 \quad \text{is equivalent to} \quad b = 2.3$$

$$\frac{6}{0} = \text{X}$$

undefined

$$6 = 0 \cdot 11 = 0$$

$$\frac{0}{0} = 0$$

undefined

$$0 = 0 \cdot 0 = 0$$

Comparison operators:

### Irrational Numbers

The set of **irrational numbers** is

{Nonrational numbers that correspond to points on the number line}.

That is, an irrational number is a number that cannot be expressed as a quotient of integers.

Irrational Numbers

.010010001...

Real Numbers

## Comparison Operators

$<$  "is less than"  $\leq$

"is less than  
or equal to"

$>$  "is greater than"  $\geq$

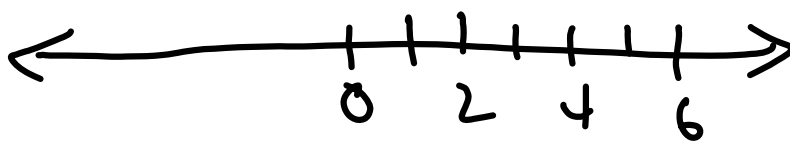
"is greater  
than or  
equal to"

Order property for Real Numbers:  
to the left of & to the right of

$$4 < 6 \quad 6 > 4$$

4 is to the left of 6

4 is less than 6



## Absolute value

Ex (p 14)

2.  $9 - 15$

4.  $2.13 - 1.13$

6.  $20 - 0$

8.  $-4 - -6$

*Are the following statements true or false?*

14.  $4 \geq 7$

*Write each sentence as a mathematical statement.*

**24.** Fifteen is greater than five.

**26.** Negative ten is less than or equal to thirty-seven.

**28.** Negative seven is not equal to seven.

*Use integers to represent the values in each statement.*

**30.** During a Green Bay Packers football game, the team gained 23 yards and then lost 12 yards on consecutive plays.

**34.** Aris Peña was deep-sea diving. During her dive, she ascended 30 feet and later descended 50 feet.

The graph below is called a bar graph. This particular graph shows the annual numbers of recreational visitors to U.S. National Parks. Each bar represents a different year, and the height of the bar represents the number of visitors (in millions) in that year.



36. What was the greatest number of visitors shown?
38. In what year(s) was the number of visitors less than 270 million?
40. Do you notice any trends shown by this bar graph?

62.  $-200$      $-20$

64.  $7.1$      $-7$

66.  $\frac{8}{2}$      $\frac{12}{3}$

68.  $|-20|$      $-200$

70.  $0$      $|0|$

72.  $\left|\frac{2}{5}\right|$      $\left|-\frac{2}{5}\right|$

74.  $-500$      $|-50|$

76.  $|-12|$      $\frac{24}{2}$

## 1.3: Fractions

Numerator & Denominator

Lowest terms

Prime numbers

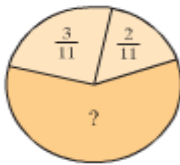
Multiplying Fractions

Dividing Fractions

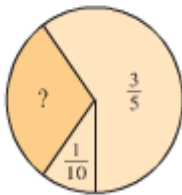
Ex: (p 22)

Find the unknown portion of the circle

56.



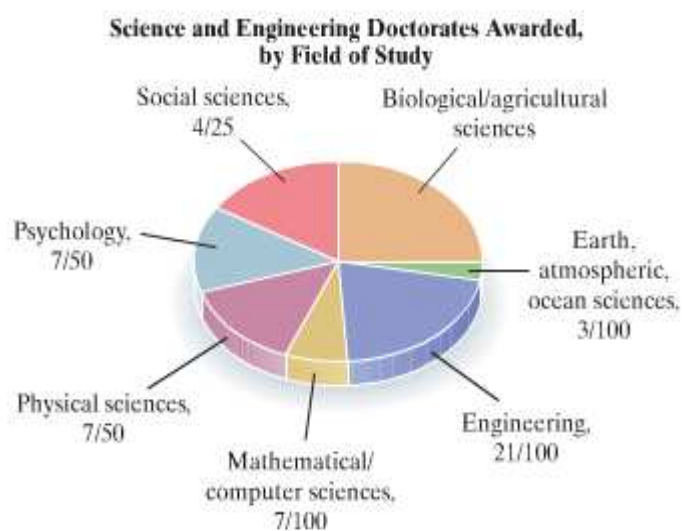
58.



60.



86. The Preakness, one of the horse races in the Triple Crown, is a  $1\frac{3}{16}$ -mile race. The Belmont, another of the three races, is  $\frac{5}{16}$  of a mile longer than the Preakness. How long is the Belmont? (*Source: Sports Illustrated*)

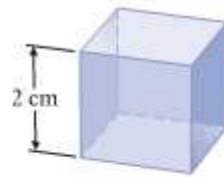


92. Engineering doctorates make up what fraction of all science and engineering doctorates awarded in the United States?
94. Social sciences and psychology doctorates together make up what fraction of all science and engineering doctorates awarded in the United States?

## 1.4: Introduction to Variable Expressions and Equations

### Exponential notation

base  $\rightarrow$   $2^3 = 2 \cdot 2 \cdot 2 = 8$  exponent  
 $2$  is a factor  $3$  times



Volume is  $(2 \cdot 2 \cdot 2)$   
cubic centimeters.

#### Order of Operations

Simplify expressions using the order below. If grouping symbols such as parentheses are present, simplify expressions within those first, starting with the innermost set. If fraction bars are present, simplify the numerator and the denominator separately.

1. Evaluate exponential expressions.
2. Perform multiplications or divisions in order from left to right.
3. Perform additions or subtractions in order from left to right.

Variable

Algebraic Expression

Equation

A solution of an equation

<i>Addition (+)</i>	<i>Subtraction (-)</i>	<i>Multiplication (·)</i>	<i>Division (÷)</i>	<i>Equality (=)</i>
Sum	Difference of	Product	Quotient	Equals
Plus	Minus	Times	Divide	Gives
Added to	Subtracted from	Multiply	Into	Is/was/should be
More than	Less than	Twice	Ratio	Yields
Increased by	Decreased by	Of	Divided by	Amounts to
Total	Less			Represents/ Is the same as

Note: subtraction & division are not commutative.

<i>Phrase</i>	<i>Translation</i>
A number decreased by 10	$x - 10$
A number subtracted from 10	$10 - x$
10 less than a number	$x - 10$
A number less 10	$x - 10$

} Notice the order.

Ex: (p 32)

66. Is 6 a solution of  $2x + 7 = 3x$ ?

68. Is 2 a solution of  $4x + 2 = x + 8$ ?

## Translate into symbols

76. One-half times a number

78. The quotient of a number and 9

82. Four subtracted from eight is equal to two squared.

92. Determine whether each is an expression or an equation.

a.  $3x^2 - 26$

b.  $3x^2 - 26 = 1$

c.  $2x - 5 = 7x - 5$

d.  $9y + x - 8$

## 1.5: Adding Real Numbers

Adding same sign

Adding different signs

Additive inverse

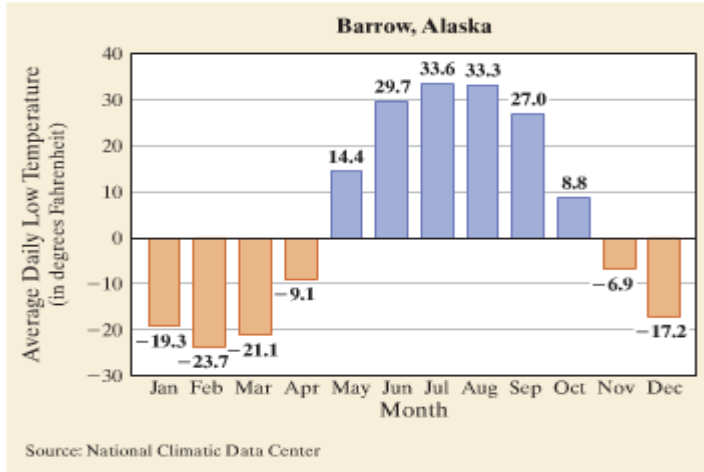
Ex: (p 40)

**58.** On January 2, 1943, the temperature was  $-4^{\circ}$  at 7:30 A.M. in Spearfish, South Dakota. Incredibly, it got  $49^{\circ}$  warmer in the next 2 minutes. To what temperature did it rise by 7:32?

**60.** The lowest point in Africa is  $-512$  feet at Lake Assal in Djibouti. If you are standing at a point 658 feet above Lake Assal, what is your elevation? (*Source: Microsoft Encarta*)

**64.** During the 2007 PGA Buick Invitational Golf Tournament, Tiger Woods won with scores of  $-6, 0, -3,$  and  $-6$ . What was his overall score? (*Source: Professional Golf Association*)

84. Is 10 a solution of  $7 = -x + 3$ ?



88. For what month is the graphed temperature the lowest?

90. For what month is the graphed temperature negative *and* closest to  $0^{\circ}$ ?

92. Find the average of the temperatures shown for the months of January, September, and October.

*If  $a$  is a positive number and  $b$  is a negative number, fill in the blanks with the words positive or negative.*

93.  $-a$  is \_\_\_\_\_.

94.  $-b$  is \_\_\_\_\_.

95.  $a + a$  is \_\_\_\_\_.

96.  $b + b$  is \_\_\_\_\_.

## 1.6: Subtracting Real Numbers

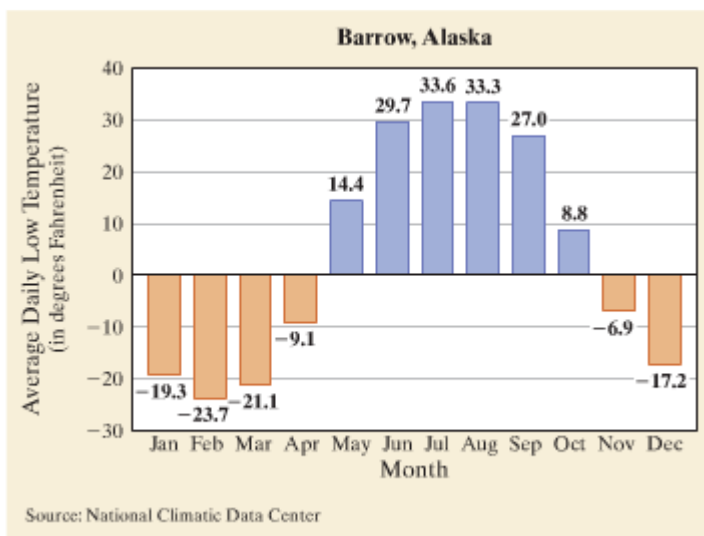
If you don't want to do subtraction

### Subtracting Two Real Numbers

If  $a$  and  $b$  are real numbers, then  $a - b = a + (-b)$ .

Ex: (p 46)

70. Much of New Orleans is below sea level. If George descends 12 feet from an elevation of 5 feet above sea level, what is his new elevation?
72. In some card games, it is possible to have a negative score. Lavonne Schultz currently has a score of 15 points. She then loses 24 points. What is her new score?
78. The airport in Bishop, California, is at an elevation of 4101 feet above sea level. The nearby Furnace Creek Airport in Death Valley, California, is at an elevation of 226 feet below sea level. How much higher in elevation is the Bishop Airport than the Furnace Creek Airport? (*Source: National Climatic Data Center*)



90. Which month had the greatest increase in temperature?
91. Which month had the greatest decrease in temperature?

## 1.7: Multiplying and Dividing Real Numbers

Two cases we'll get to  
Same sign & Different sign

### Zero as a Factor

If  $b$  is a real number, then  $b \cdot 0 = 0$ . Also,  $0 \cdot b = 0$ .

### ► Helpful Hint

You may have noticed from the example that if we multiply:

- an *even* number of negative numbers, the product is *positive*.
- an *odd* number of negative numbers, the product is *negative*.

### Helpful Hint

Be careful when identifying the base of an exponential expression.

$$\begin{array}{ccc} (-3)^2 & & -3^2 \\ \text{Base is } -3 & & \text{Base is } 3 \\ (-3)^2 = (-3)(-3) = 9 & & -3^2 = -(3 \cdot 3) = -9 \end{array}$$

### Reciprocals or Multiplicative Inverses

Two numbers whose product is 1 are called reciprocals or multiplicative inverses of each other.

### Quotient of Two Real Numbers

If  $a$  and  $b$  are real numbers and  $b$  is not 0, then

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$$

### Multiplying and Dividing Real Numbers

1. The product or quotient of two numbers with the *same* sign is a positive number.
2. The product or quotient of two numbers with *different* signs is a negative number.

**Zero as a Divisor or Dividend**

1. The quotient of any nonzero real number and 0 is undefined. In symbols, if  $a \neq 0$ ,  $\frac{a}{0}$  is **undefined**.
2. The quotient of 0 and any real number except 0 is 0. In symbols, if  $a \neq 0$ ,  $\frac{0}{a} = 0$ .

Ex: (p 56)

If  $x = -5$  and  $y = -3$ , evaluate each expression.

104.  $4x + 5y$

110.  $\frac{2y - 12}{x - 4}$

114. At the end of the third quarter of 2006, General Motors reported a net loss of \$115 million. If this continued, what would General Motor's income be after four more quarters?  
(Source: General Motors)

116. Is  $-4$  a solution of  $2x = x - 1$ ?

120. Is  $-4$  a solution of  $2x + 4 = x + 8$ ?

*Write each of the following as an expression and evaluate.*

**132.** The sum of 1 and the product of  $-8$  and  $-5$

**134.** 7 subtracted from the quotient of 0 and 5

## 1.8: Properties of Real Numbers

### Commutative Properties

**Addition:**  $a + b = b + a$

**Multiplication:**  $a \cdot b = b \cdot a$

### Associative Properties

**Addition:**  $(a + b) + c = a + (b + c)$

**Multiplication:**  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

### Distributive Property of Multiplication Over Addition

$$a(b + c) = ab + ac$$

**Identities for Addition and Multiplication**

0 is the identity element for addition.

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

1 is the identity element for multiplication.

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a$$

**Additive or Multiplicative Inverses**

The numbers  $a$  and  $-a$  are additive inverses or opposites of each other because their sum is 0; that is,

$$a + (-a) = 0$$

The numbers  $b$  and  $\frac{1}{b}$  (for  $b \neq 0$ ) are reciprocals or multiplicative inverses of each other because their product is 1; that is,

$$b \cdot \frac{1}{b} = 1$$

Ex: (p 63)

Simplify

36.  $5(7 + 8y)$

38.  $3(8x - 1)$

40.  $2(x + 5)$

42.  $-3(z - y)$

44.  $-5(2r + 11)$

52.  $-(q - 2 + 6r)$

56.  $-\frac{1}{5}(10a - 25b)$

60.  $-11(5x + 3) + 10$