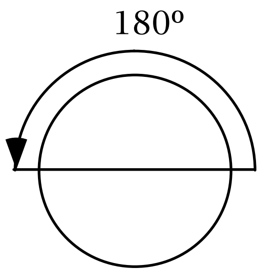
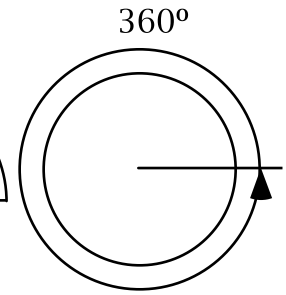
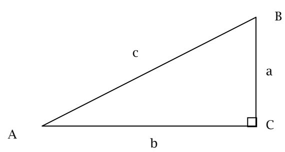
An Introduction to Trigonometry

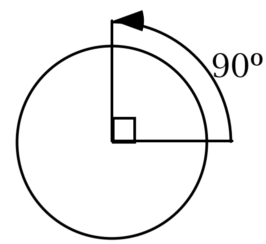
Trigonometry is the study of angles, often within a right triangle, and the ratio of the sides of the right triangles.

Perhaps the appropriate place to start is to discuss a way of measuring angles. One system is by degrees. I will define a full circle (or one complete revolution) as 360 degrees (written 360º). therefore, one–half of a circle is 180º, one–quarter is 90º, etc.. Notice, in the diagrams to the right, I have drawn circles, not triangles.









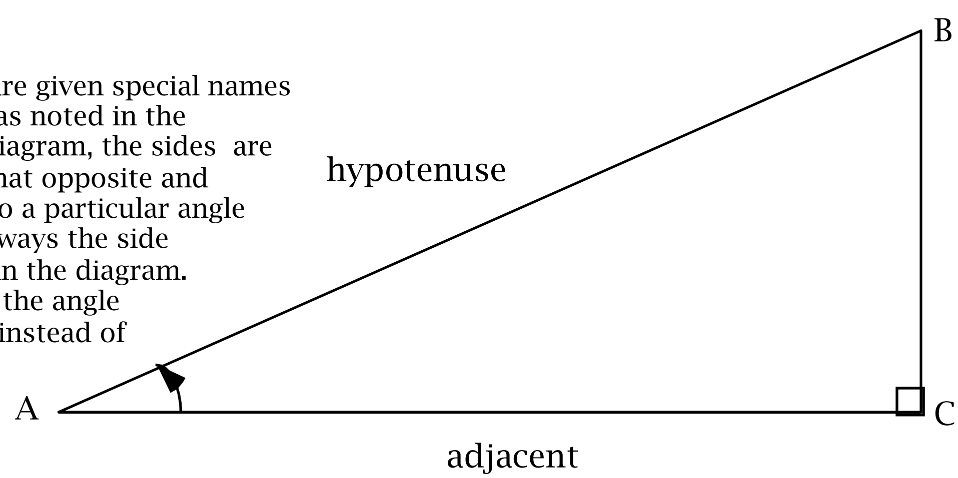
If I draw a triangle, it would have angles in it. The diagram above shows angles A, B, and C. I will denote angle A by ∠A, and B by ∠B, and angle C by ∠C. The side opposite ∠A is denoted by a. By convention angles are denoted by capital letters and sides by small letters. Some books use Greek letters to denote angles, e.g. α, β, and γ; I will not use them to avoid potential and unnecessary confusion.

The angles in a triangle fall into three categories: acute (less than 90º), right (exactly 90º), and obtuse (between 90º and 180º). Another kind of angle you might talk about is a straight angle (exactly 180º).

I will be talking exclusively about one kind of triangle, a right triangle. That is, a triangle which has a right angle (90º angle) in it.

<u>N.B.</u> Right angles are usually denoted in a diagram by a box–like figure, such as in the diagrams above and below..

The sides of a right triangle are given special names relative to a particular angle as noted in the diagram to the right. In the diagram, the sides are named relative to ∠A. Note that opposite and adjacent are always relative to a particular angle whereas the hypotenuse is always the side opposite the right angle, ∠C in the diagram. Job for you: label the sides if the angle you want to talk about is ∠B instead of ∠A.



As I said at the beginning, trigonometry is the study of ratios of the sides of a right triangle. In order to make life easier for us, generally I will call the angle I want to discuss ∠A. Also, I will normally have that angle in what is called the standard position [see ∠A in the diagrams above and below]. Now, the various ratios are described below with a special name assigned to each.*

sine A	= sin A	= $\frac{\text{opposite}}{\text{hypotenuse}}$	cosecant A	= csc A	= $\frac{\text{hypotenuse}}{\text{opposite}}$
cosine A	= cos A	= $\frac{\text{adjacent}}{\text{hypotenuse}}$	secant A	= sec A	= $\frac{\text{hypotenuse}}{\text{adjacent}}$
tangent A	= tan A	= $\frac{\text{opposite}}{\text{adjacent}}$	cotangent A	= cot A	= $\frac{\text{adjacent}}{\text{opposite}}$

A mnemonic device some students find helpful is Chief SOHCAHTOA.
Note: sine and cosecant are reciprocals of each other.
cosine and secant are reciprocals of each other.
tangent and cotangent are reciprocals of each other.

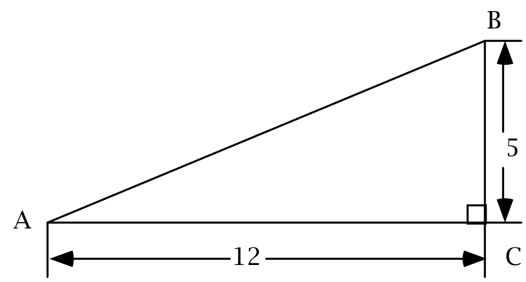
Example 1: For the following triangle find all six trig functions for: (i) angle A (ii) angle B.

First, I must find c (the side opposite ∠C).
By the Pythagorean Theorem,

$$a^2 + b^2 = c^2$$

In this problem, that means

$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$169 = c^2$$
$$c = 13.$$



Now I am ready to find the trig functions:

(i) $\sin A = \frac{5}{13}$ $\csc A = \frac{13}{5}$

$\cos A = \frac{12}{13}$ $\sec A = \frac{13}{12}$

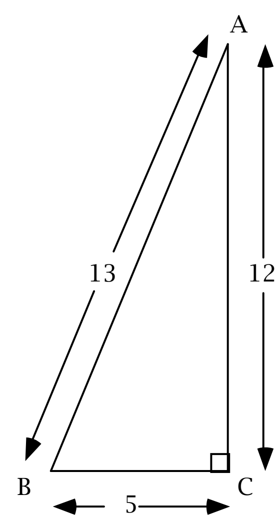
$\tan A = \frac{5}{12}$ $\cot A = \frac{12}{5}$

(ii) $\sin B = \frac{12}{13}$ $\csc B = \frac{13}{12}$

$\cos B = \frac{5}{13}$ $\sec B = \frac{13}{5}$

$\tan B = \frac{12}{5}$ $\cot B = \frac{5}{12}$

<u>N.B.:</u> $\angle A + \angle B = 90^o$



* *Sine* is a mistranslation of the Arabic for a half-chord. *Tangent* is a word describing a line that 'touches' a circle. And *Secant* comes from the Latin secare which means to cut. The derivation of the co-'s will be explained later.

Notice in the last example that I found the various trig functions even though I didn't know what ∠A and ∠B equaled. Suppose I know what an angle equals. Can I find sine, cosine, etc.? The answer is yes. There are several ways to find the trig functions. One of the oldest is by trig tables. There is a trig table in the back of this booklet that you may use. Today most people use a calculator. The most I can say is to read the booklet that came with your calculator to determine how to use it.

Trig tables can be used to both find the various trig functions of an angle and to find the angle given a particular ratio. See some examples below.

Example 2:

Finding the trig functions:

sin 13° = *0.2250* tan 44° = *0.9657*

cos 27°= *0.8910* cot 19° = *2.904*

Finding Angles:

cos A= 0.4695 *means* *∠A = 62° find .4695 in the cos col and read the angle*

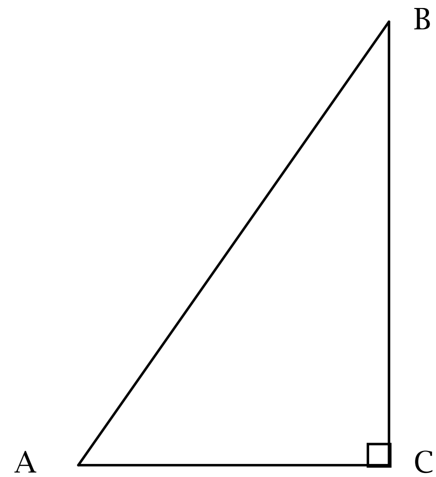
sin B = 0.4848 *means* *∠B = 29° find .4848 in the sin col and read the angle*

tan D= 14.30 *means* *∠D = 86°*

cot F = 0.5095 *means* *∠F = 63°*

<u>N.B.</u> For our purposes, if a given trig value is not in the table, pick the angle whose trig value is closest to the given value. If you are using a calculator, find the angle to the nearest tenth of a degree (remember to round).

I will now give several examples of finding sides and angles of triangles using the trig functions.



Example 3:

I will assume ∠C = 90°. Given a right triangle with ∠A = 42° and a = 10.
Find: ∠B, ∠C, b, and c.

I will first find the angles:

Since ∠A = 42° and ∠C = 90° and the angles of any triangle add up to 180°,
∠B = 180° – (42° + 90°) = 48°.
Note I could have said that ∠A and ∠B must add up to 90°, since ∠C takes the other 90°, so ∠B = 90° – 42°.

Next I want to find sides b and c. I know

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}\text{, where } \angle A = 42^\circ \text{ and } a = 10. \text{ Thus}$$

$$\tan 42 = \frac{10}{b} \quad \text{or}$$

$$0.9004 = \frac{10}{b}$$

$$0.9004b = 10$$

$$b = \frac{10}{0.9004}$$

$$b \approx 11.1$$

Note: generally I will find a side to the nearest tenth. and will dispense with the ≈, which means approximately equal to.

I can find ***c*** *by several methods. I could use sine, cosine, or the Pythagorean Theorem.*

Method 1: Using sine, I would have

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}\ , \text{ where } \angle A = 42^o \text{ and } a = 10.$$

Thus:

$$\sin 42^o = \frac{10}{c}$$
$$0.6691 = \frac{10}{c}$$
$$0.6691c = 10$$
$$c = 14.9$$

Method 2: Using the Pythagorean Theorem, I know

$$a^2 + b^2 = c^2\ , \text{ where } a = 10 \ \&\ b = 11.1$$

Thus

$$10^2 + 11.1^2 = c^2$$
$$100 + 123.21 = c^2$$
$$c^2 = 223.21$$
$$c = 14.9$$

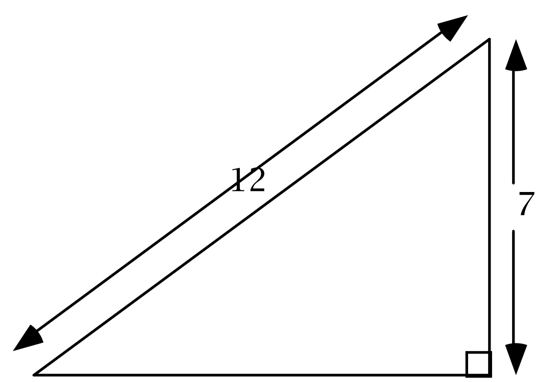
Method 3: Using cosine would be similar to using the sine.

Thus for this triangle I have:

$\angle A = 42^o$	$a = 10$
$\angle B = 48^o$	$b = 11.1$
$\angle C = 90^o$	$c = 14.9$

Note: Almost all trig function values are approximations. Your table is only accurate to 4 digits, and your calculator to 8–12, depending on the calculator.

Example 4: Find all the sides and angles of the given right triangle where $a = 7$ & $c = 12$.



It would probably be easiest to find b first using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$7^2 + b^2 = 12^2$$
$$49 + b^2 = 144$$
$$b^2 = 95$$
$$b = 9.7$$

Remember, I assumed $\angle C = 90^o$, so I only need to find $\angle A$ and $\angle B$.

I know

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} = \frac{7}{12} \approx 0.5833$$

Thus: $A = 36^o$.

Now I know $A + B = 90^o$

Thus: $B = 90^o - 36^o = 54^o$

To summarize:

$\angle A = 36^o$	$a = 7$
$\angle B = 54^o$	$b = 9.7$

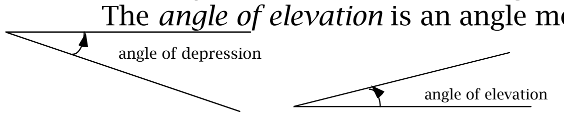
$\angle C \;=\; 90^o \qquad\qquad c \;=\; 12$

Word Problems

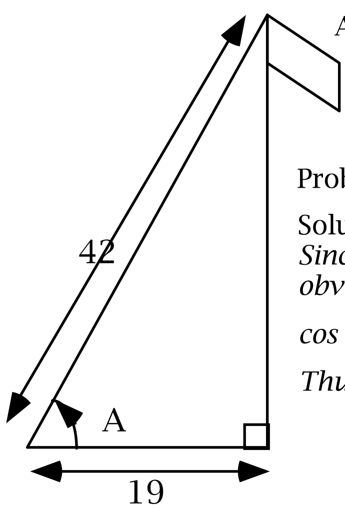
The general technique to use in solving word problems is:

1. set up a right triangle using the given information;
2. identify the unknown quantity (ies);
3. analyze to determine which trig function(s) to use;
4. set up a trig equation;
5. solve; and
6. check.

Note: The *angle of depression* is an angle measured from the horizontal down
The *angle of elevation* is an angle measured from the horizontal up.



Example 5:



A 42 foot long rope is required to reach from the top of a flagpole to a point 19 feet from the foot of the pole. Assuming the ground is level, what angle does the rope make with the ground?

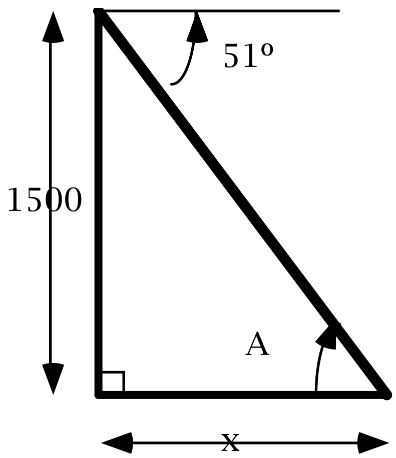
Problem: *Find A (in the diagram)*

Solution:
Since I know the adjacent (19) and the hypotenuse (42), the cosine would be the obvious choice.

$$\cos A \;=\; \frac{adjacent}{hypotenuse} \;=\; \frac{19}{42} \;=\; 0.4524$$

Thus: $\angle A \;=\; 63^o$

Example 6:



From a vertical height of 1500 m, a balloonist notes that the angle of depression to the enemy trench is 51º. Find the distance from the trench to a point on the level ground directly below the balloonist.

Problem: *Find x (in the given diagram).*

Solution:
Since the angle of depression is 51º, ∠B = 39º (why?)
Thus ∠A = 51º, so

$$\tan A = \frac{opposite}{adjacent}$$

$$\tan 51^\circ = \frac{1500}{x}$$

$$1.235 = \frac{1500}{x}$$

$$1.235x = 1500$$

$$x = \frac{1500}{1.235}$$

$$x = 1214.6 \quad m$$

Trigonometry on The Cartesian Co–ordinate System

If you remember when I first discussed angles, I drew circles and talked about angles larger than 90º. But, so far, I have only discussed angles 90º or less. I will now discuss any angle. In order to facilitate this, I will introduce the concept of *an angle in standard position.*

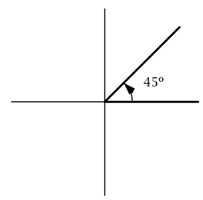
An angle is said to be in **standard position** if:

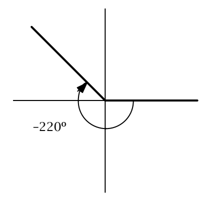
1. its **vertex** is at the origin, and
2. its **initial side** is the positive x–axis.

Other side of the angle is called the **terminal side**.

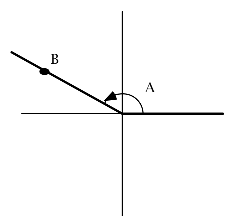
By convention, angles measured counterclockwise are considered positive, while angles measured clockwise are considered negative.

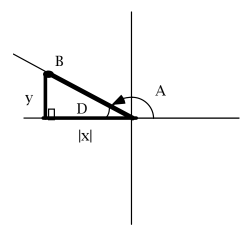
Example 7:





An obvious problem with our trig table is that it goes no further than 90º. In fact, no trig table will have more than 90º. Some will only have 45º; can you figure out how to get the other 45º up to 90º? (Hint: look at the sin 30º and the cosine 60º).



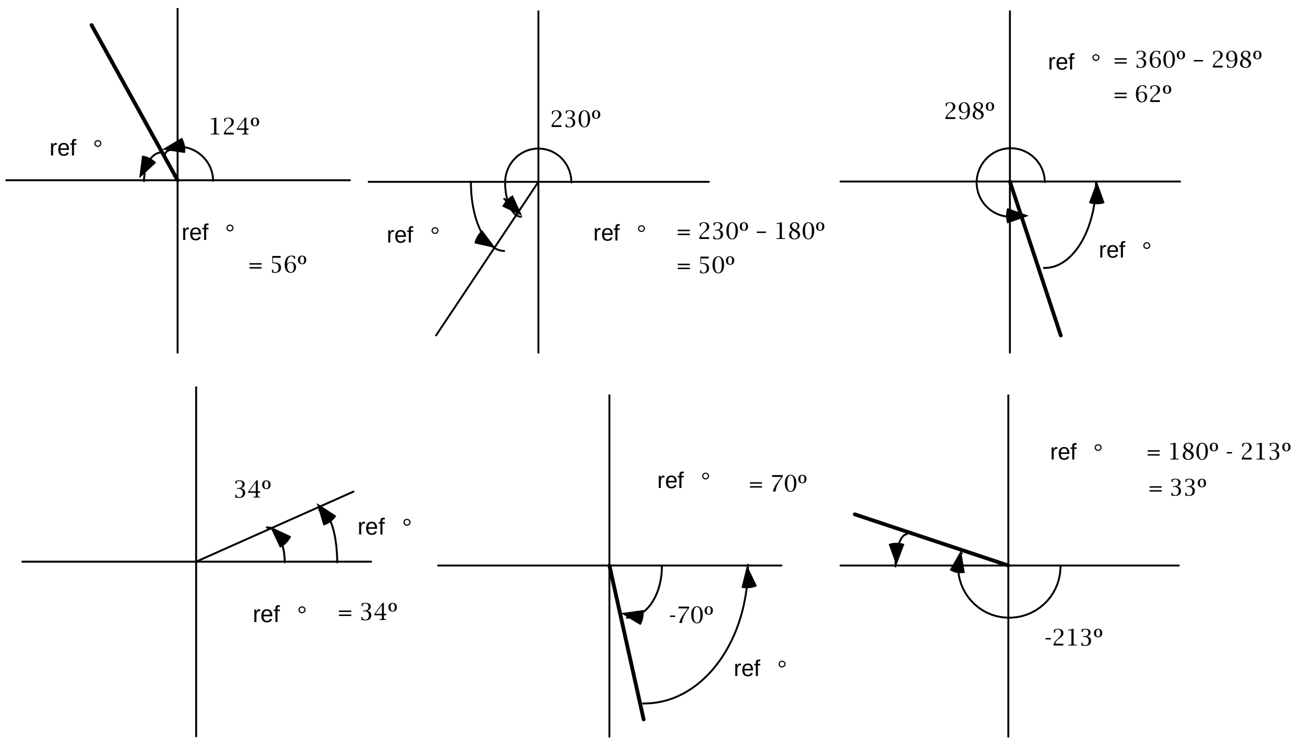


To show you why a table need only list 0º–90º, consider the angle in the diagram. If you wanted to find sin A, remember we normally have a ratio of sides of a triangle. Pick a point on the terminal side of A, B. B has co–ordinates (x,y). Thus we could construct a triangle with sides |x| and y (remember x is negative, but distance is always positive). This happens to be the triangle associated with ∠A. Notice the angle inside the triangle which is in the position we usually had the angle we wanted to discuss (∠D). How do ∠A and∠D relate?

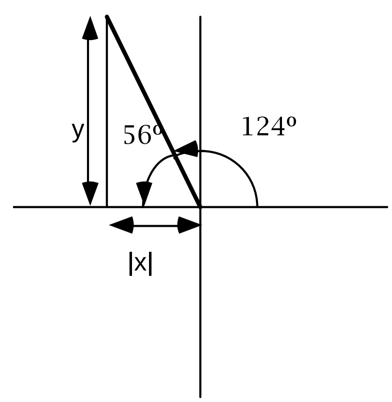
$$\angle D = 180^\circ - \angle A$$

Also notice ∠D is less than 90º and greater than 0º. We will call ∠D ∠A's **reference angle**. In general, the reference angle of any angle is the angle (between 0º and 90º) the original angle's terminal side makes with the x–axis.

Example 8:



By using reference angles, a trig table need not go further than 0º to 90º. However, even though the reference angle of 124º is 56º, cos 124 $\neq$ cos 56º. Oops, what happened? Reference angles were supposed to help. Look at 124º again. Pick a point P(x,y) on the terminal side. Drop a vertical line to the X–axis. You now have a right triangle. Its sides are |x| (remember x is negative) , y, and r ($r = \sqrt{x^2+y^2}$ [Pythagorean Theorem]).



Note: x is <u>negative</u>. Now:

$$\cos 56^\circ = \frac{|x|}{r}$$

I hope that's no problem; remember $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Find cos 124º on a calculator (or trust me). It's the same as the cos 56º, except for the negative sign. Since x was negative, consider the following definition:

$$\cos 124^\circ = \frac{x}{r} .$$

Since x is actually negative (and r is always positive) this is negative, and it is equal to cos 56º except for the sign. Therefore, the reference angle of 124º, 56º, will tell you magnitude just not the sign of its trig functions.

Thus we can apparently use the reference angle to find the trig function of any angle, but we still need a way to find the sign.

Let's stop for a minute and go back over our definitions of the various trig functions for an angle in the first quadrant, <u>i.e.</u> 0º to 90º. Pick any point on the terminal side, P(x,y). Using this point , the radius, and the X–axis we can construct the triangle shown with sides as shown. Using the definitions of the various trig functions, we get the following equations:

$\sin A = \frac{y}{r}$ $\csc A = \frac{r}{y}$

$\cos A = \frac{x}{r}$ $\sec A = \frac{r}{x}$

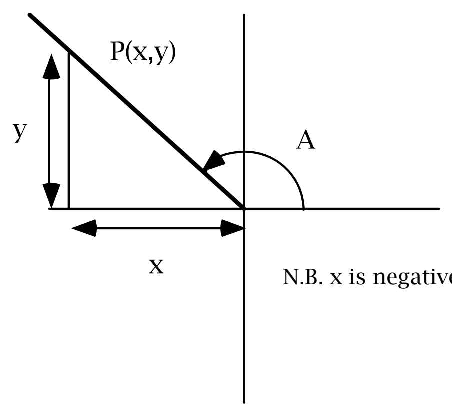
$\tan A = \frac{y}{x}$ $\cot A = \frac{x}{y}$

These equations are appropriate, regardless of A.
Thus if ∠A is as in the diagram

$\sin A = \frac{y}{r}$ is positive,

$\cos A = \frac{x}{r}$ is negative, and

$\tan A = \frac{y}{x}$ is negative



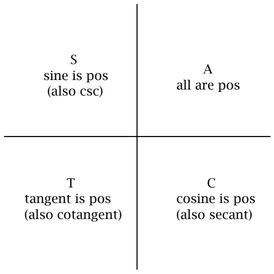
since x is negative. Note: r is *always* considered positive. We could come up with the following chart to help us remember whether a trig function is positive or negative.

$x < 0$, $y > 0$	$x > 0$, $y > 0$
$x < 0$, $y < 0$	$x > 0$, $y < 0$

therefore

$\sin A > 0$, $\cos A < 0$, $\tan A < 0$	$\sin A > 0$, $\cos A > 0$, $\tan A > 0$
$\sin A < 0$, $\cos A < 0$, $\tan A > 0$	$\sin A < 0$, $\cos A > 0$, $\tan A < 0$

A mnemonic some students find helpful is: "**A**ll **S**tudents **T**ake **C**ourses"

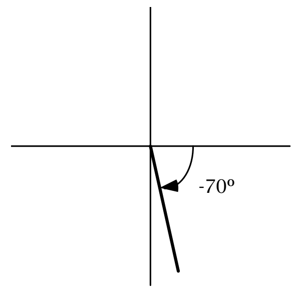


Example 9:

(i) Sketch –70º in standard position

(ii) Find the reference angle for –70º

(iii) Find sin(–70º) and cos(–70º)



Solution:

(i) See the sketch to the right.

(ii) The reference angle is 70º

(iii) $\sin(-70^\circ) = -\sin(70^\circ) = -0.9397$

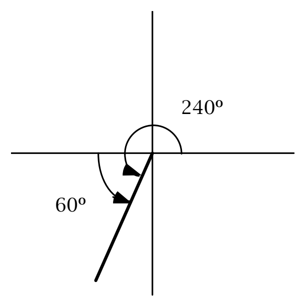
$\cos(-70^\circ) = \cos(70^\circ) = 0.3420$

Example 10:

(i) Sketch 240º in standard position

(ii) Find the reference angle for 240º

(iii) Find the tan(240º) and csc(240º)



Solution:

(i) See sketch to the above

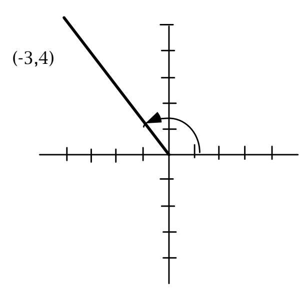
(ii) The reference angle is 60º (240º – 180º = 60º)

(iii) $\tan(240^\circ) = +\tan(60^\circ) = 1.732$

$$\csc(240^\circ) = -\csc(60^\circ) = -\frac{1}{\sin(60^\circ)} = -\frac{1}{0.8660} = -1.155$$

Example 11:

Use our new definitions of the trig functions to find all the trig function of the angle whose terminal side includes the point
(–3,4)



We know x = –3 and y = 4
and by the Pythagorean Theorem

$$\begin{aligned} r^2 &= x^2 + y^2 \\ &= (-3)^2 + 4^2 \\ &= 25 \\ r &= 5 \end{aligned}$$

Thus, the trig functions are:

$\sin A = \frac{4}{5}$ $\qquad \csc A = \frac{5}{4}$

$\cos A = \frac{-3}{5} = -\frac{3}{5}$ $\qquad \sec A = \frac{5}{-3} = -\frac{5}{3}$

$\tan A = \frac{4}{-3} = -\frac{4}{3}$ $\qquad \cot A = \frac{-3}{4} = -\frac{3}{4}$

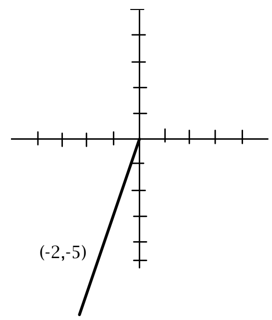
Example 12: (same as Ex 11) for (–2,–5)

We know:

$x = -2$
$y = -5$

and

$r^2 = x^2 + y^2$
$r^2 = (-2)^2 + (-5)^2$
$= 29$
$r = \sqrt{29}$



Thus, the trig functions are

$\sin A = \frac{-5}{\sqrt{29}} = -\frac{5}{\sqrt{29}}$ $\qquad \csc A = \frac{-\sqrt{29}}{5} = -\frac{\sqrt{29}}{5}$

$\cos A = \frac{-2}{\sqrt{29}} = -\frac{2}{\sqrt{29}}$ $\qquad \sec A = \frac{-\sqrt{29}}{2} = -\frac{\sqrt{29}}{2}$

$\tan A = \frac{-5}{-2} = \frac{5}{2}$ $\qquad \cot A = \frac{-2}{-5} = \frac{2}{5}$

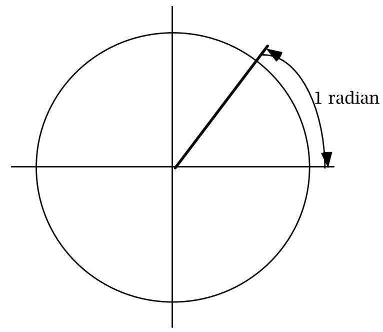
Other Ways of Measuring Angles

So far we have measured all angles in degrees. There are several other ways to measure angles; the other most prevalent way is radians. Radians are a very important way to measure angles. The word radian comes from the word **radius** because a radian is an arc of length one radius. Imagine a piece of string one radius long laid along the edge of a circle. The arc you have subtends (a fancy math word meaning cuts off) an angle of 1 radian.

One radian equals approximately 57º18'. (Note 18' means 18 minutes. There are 60 minutes in 1 degree). Recall from plane geometry, the formula for the equation of the circumference:

$C = 2\pi r$

r : radius
C : circumference of the circle
π : an irrational number approximately equal to 3.1415926536



Thus, there are 2π radians in one revolution, π in one-half revolution, $\frac{\pi}{2}$ in one-quarter revolution, and so on.

We also know 360º is one revolution, 180º is one-half revolution, 90º is one-quarter, etc. Therefore:

$360^o = 2\pi$ radians $\qquad 180^o = \pi$ radians $\qquad 90^o = \frac{\pi}{2}$ radians

Note: radians are often abbreviated R or Rad. The most common abbreviation is nothing, which is what I will use. It would be convenient if we had a general conversion formula for radians to degrees and back. The easiest way to do this is to create a proportion, such as

$$\frac{x}{y^o} = \frac{\pi}{180^o}$$

This can be solved for x or yº:

$$x = y\left(\frac{\pi}{180^o}\right) \qquad x\left(\frac{180^o}{\pi}\right) = y^o$$

Example 13:

(i) Convert 45º to radians

$$\frac{x}{45^o} = \frac{\pi}{180^o}$$
$$x = \frac{45\pi}{180}$$
$$x = \frac{\pi}{4}$$

(ii) convert $\frac{3\pi}{5}$ to degrees

$$\frac{\frac{3\pi}{5}}{y} = \frac{\pi}{180^o}$$
$$\frac{3\pi}{5} = \frac{\pi}{180^o}\, y$$
$$y = \frac{3\pi}{5} \exists \frac{180^o}{\pi} = 108^o$$

Trig Problems:

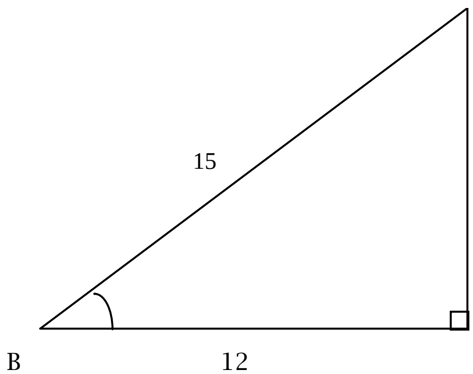
1. Evaluate the following. If you use a calculator, round your answers to 4 decimal places.

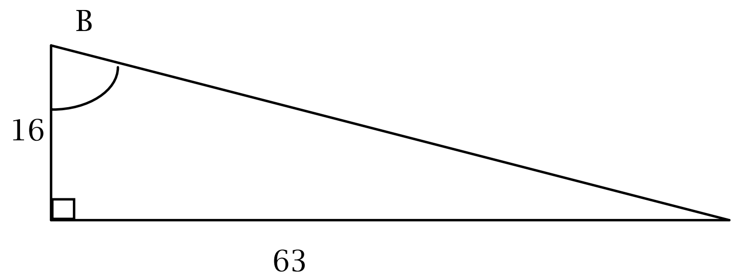
a) sin 10°	b) cos 43°	c) tan 26°
d) cot 40°	e) sin 83°	f) cos 71°
g) tan 73°	h) cot 51°	

2. Find angle A. If you are using a calculator, round your answers to the nearest tenth.

a) sin A = 0.8988	b) cos A = 0.6018	c) tan A = 2.1445
d) cot A = 0.8693	e) sin A = 0.9755	f) cos A = 0.1754
g) tan A = 0.1998	h) cot A = 0.8785	

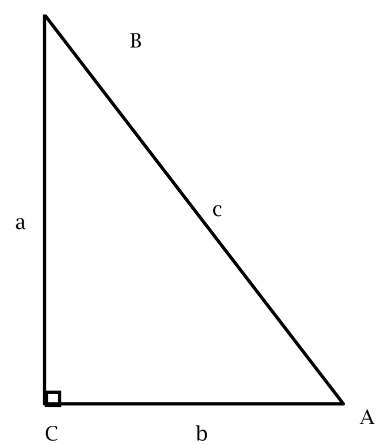
3. find the six trigonometric functions for angle B.





4. Solve each of the following right triangles. If you use a calculator, find the angles to the nearest tenth of a degree. Find the lengths of sides to the nearest tenth.

a)	A	=	36.0°	a	=	27.2
b)	B	=	13.0°	b	=	98.1
c)	A	=	17.0°	b	=	13.6
d)	B	=	23.0°	a	=	34.5
e)	A	=	48.0°	c	=	48.3
f)	B	=	82.0°	c	=	98.2
g)	a	=	12.0	b	=	18.0
h)	a	=	16.0	c	=	20.0
i)	b	=	18.0	c	=	40.0



5. Solve each of the following:
   a) A guy wire to a pole makes an angle of 73.0º with the level ground and is 14.5 feet from the pole at the ground. How far above the ground is the wire attached to the pole?

   b) A road rises 3 feet per 100 horizontal feet. What angle does it make with the horizontal?

   c) What is the angle of elevation of the sun when a 6.0 foot man casts a 10.3 foot shadow?

   d) From a balloon 2500 feet high, a command post is seen with an angle of depression of 8.0º. How far is it from a point on the ground below the balloon to the command post? (Round to the nearest ten. feet)

   e) The angle of elevation of the top of a building from a point at street level 200 feet from the base of the building is 48.0º. How high is the building?

   f) Ship A is due west of lighthouse L. Ship B is 12 miles south of A. Angle B is 63.0º. How far is ship B from the lighthouse?

   g) From the top of a hill 2 miles high, the angles of depression to two towns in a line with the hill are 81.0º and 14.0º. How far apart are the towns? (Warning there are two sets of answers).

   h) A man 30 feet above the ground on a telephone pole sights his car 20 feet from the base of the pole. What is the angle of depression to the car?

   i) A 45 foot guy wire is attached to a pole 25 feet above the ground. What angle does the wire make with the ground? (The wire gores from the ground to the pole)

   j) A 25.0 foot ladder is leaning against the side of a house. If the ladder makes a 66.0º angle with the ground, how far from the house is the foot of the ladder?

   k) A weather balloon is directly west of two observation stations 10 miles apart. The angles of elevation of the balloon from the two stations are 18.0º and 78.0º. How high is the balloon?

6. For each of the following:
   i) Sketch the angle in standard position;
   ii) Find its reference angle R; and
   iii) Evaluate the given trig function.

a)	115º,	sin(115º)	b)	–120º,	cos(–120º)	c)	170º,	tan(170º)
d)	–30º,	sin(–30º)	e)	300º,	cos(300º)	f)	–110º,	tan(–110º)

7. Find the six trig function values of the angle A in standard position having terminal side which passes through the given point.
   a) (–8,6) b) (–7,24) c) (5,–5)

8. Express in radian measure:
   a) 45° b) -90° c) 144°
   d) 330°

9. Express in degree measure:
   a) $\frac{\pi}{3}$ b) $\frac{-\pi}{4}$ c) $\frac{7\pi}{10}$
   d) $\frac{-5\pi}{6}$

Trig Problems Answers

1.
   a) 0.1736 b) 0.7314 c) 0.4877
   d) 1.1918 e) 0.9925 f) 0.3256
   g) 3.2709 h) 0.8098

2.
   a) 64.0° b) 53.0° c) 65.0°
   d) 49.0° e) 77.3° f) 79.9°
   g) 11.3° h) 48.7°

3.
   a) $\sin B = \frac{3}{5}$ $\csc B = \frac{5}{3}$

   $\cos B = \frac{4}{5}$ $\sec B = \frac{5}{4}$

   $\tan B = \frac{3}{4}$ $\cot B = \frac{4}{3}$

   b) $\sin B = \frac{63}{65}$ $\csc B = \frac{65}{63}$

   $\cos B = \frac{16}{65}$ $\sec B = \frac{65}{16}$

   $\tan B = \frac{63}{16}$ $\cot B = \frac{16}{63}$

4.

a) $B = 54.0^\circ$, $b = 37.4$, $c = 46.2$

b) $A = 77.0^\circ$, $a = 424.9$, $c = 436.1$

c) $B = 73.0^\circ$, $a = 4.2$, $c = 14.2$

d) $A = 67.0^\circ$, $b = 14.6$, $c = 37.5$

e) $B = 42.0^\circ$, $a = 35.9$, $b = 32.3$

f) $A = 8.0^\circ$, $a = 14.0$, $b = 97.2$

g) $A = 33.7^\circ$, $B = 56.3^\circ$, $c = 21.6$

h) $A = 53.1^\circ$, $B = 36.9^\circ$, $b = 12.0$

i) $A = 63.3^\circ$, $B = 26.7^\circ$, $a = 35.7$

5. Note: Your answers may vary slightly from mine depending on rounding

a) 47.4 ft
b) 1.7°
c) 30.2°
d) 177,788.4 ft
e) 222.1 ft
f) 26.4 mi
g) 8.3 mi or 7.7 mi
h) 56.3°
i) 33.7°
j) 10.2 ft
k) 3.5 mi

6.

a) ref $\angle = 65^\circ$, $\sin(115^\circ) = 0.9063$

b) ref $\angle = 60^\circ$, $\cos(-120^\circ) = -0.5000$

c) ref $\angle = 10^\circ$, $\tan(170^\circ) = -0.1763$

d) ref $\angle = 30^\circ$, $\sin(-30^\circ) = -0.5000$

e) ref $\angle = 60^\circ$, $\cos(300^\circ) = 0.5000$

f) ref $\angle = 70^\circ$, $\tan(-110^\circ) = 2.7475$

7.

a) $\sin A = \frac{3}{5}$ $\quad \csc A = \frac{5}{3}$

$\cos A = -\frac{4}{5}$ $\quad \sec A = -\frac{5}{4}$

$\tan A = -\frac{3}{4}$ $\quad \cot A = -\frac{4}{3}$

b) $\sin A = -\frac{24}{25}$ $\quad \csc A = -\frac{25}{24}$

$\cos A = -\frac{7}{25}$ $\quad \sec A = -\frac{25}{7}$

$\tan A = \frac{24}{7}$ $\quad \cot A = \frac{7}{24}$

c) $\sin A = -\frac{1}{\sqrt{2}}$ $\quad \csc A = -\sqrt{2}$

$\cos A = \frac{1}{\sqrt{2}}$ $\quad \sec A = \sqrt{2}$

$\tan A = -1$ $\quad \cot A = -1$

Table of Trigonmetric Functions

Angle in Degrees	sin	cos	tan		
0º	0.0000	1.000	0.0000	——	90º
1º	0.0175	0.9998	0.0175	57.29	89º
2º	0.0349	0.9994	0.0349	28.64	88º
3º	0.0523	0.9986	0.0524	19.08	87º
4º	0.0698	0.9976	0.0699	14.30	86º
5º	0.0872	0.9962	0.0875	11.43	85º
6º	0.1045	0.9945	0.1051	9.514	84º
7º	0.1219	0.9925	0.1228	8.144	83º
8º	0.1392	0.9903	0.1405	7.115	82º
9º	0.1564	0.9877	0.1584	6.314	81º
10º	0.1736	0.9848	0.1763	5.671	80º
11º	0.1908	0.9816	0.1944	5.145	79º
12º	0.2079	0.9781	0.2126	4.705	78º
13º	0.2250	0.9744	0.2309	4.331	77º
14º	0.2419	0.9703	0.2493	4.011	76º
15º	0.2588	0.9659	0.2679	3.732	75º
16º	0.2756	0.9613	0.2867	3.487	74º
17º	0.2924	0.9563	0.3057	3.271	73º
18º	0.3090	0.9511	0.3249	3.078	72º
19º	0.3256	0.9455	0.3443	2.904	71º
20º	0.3420	0.9397	0.3640	2.747	70º
21º	0.3584	0.9336	0.3839	2.605	69º
22º	0.3746	0.9272	0.4040	2.475	68º
23º	0.3907	0.9205	0.4245	2.356	67º
24º	0.4067	0.9135	0.4452	2.246	66º
25º	0.4226	0.9063	0.4663	2.145	65º
26º	0.4384	0.8988	0.4877	2.050	64º
27º	0.4540	0.8910	0.5095	1.963	63º
28º	0.4695	0.8829	0.5317	1.881	62º
29º	0.4848	0.8746	0.5543	1.804	61º
30º	0.5000	0.8660	0.5774	1.732	60º
31º	0.5150	0.8572	0.6009	1.664	59º
32º	0.5299	0.8480	0.6249	1.600	58º
33º	0.5446	0.8387	0.6494	1.540	57º
34º	0.5592	0.8290	0.6745	1.483	56º
35º	0.5736	0.8192	0.7002	1.428	55º
36º	0.5878	0.8090	0.7265	1.376	54º
37º	0.6018	0.7986	0.7536	1.327	53º
38º	0.6157	0.7880	0.7813	1.280	52º
39º	0.6293	0.7771	0.8098	1.235	51º
40º	0.6428	0.7660	0.8391	1.192	50º
41º	0.6561	0.7547	0.8693	1.150	49º
42º	0.6691	0.7431	0.9004	1.111	48º
43º	0.6820	0.7314	0.9325	1.072	47º
44º	0.6947	0.7193	0.9657	1.036	46º
45º	0.7071	0.7071	1.000	1.000	45º
	cos	**sin**		**tan**	**Angle in Degrees**