$$x^2 + 2ax + a^2 = (x+a)^2$$

Converting

$$x^2 + 6x + 3^2 = (x+3)^2$$

$$x^2 - 10x + (-5)^2 = (x-5)^2$$

$$x^2 + 8x + (4)^2 = (x+4)^2$$

$$x^2 - 24x + (-12)^2 = (x-12)^2$$

$$x^2 + 6x + 2 = 0$$

$$x^2 + 6x + 3^2 = -2 + 9$$

$$(x+3)^2 = 7$$

$$x+3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

$$x^2 - 10x - 4 = 0$$

$$x^2 - 10x + (-5)^2 = 4 + 25$$

$$(x-5)^2 = 29$$

$$x-5 = \pm\sqrt{29}$$

$$x = 5 \pm \sqrt{29}$$

9.2: Solving Quadratic Equations by Completing the Square

Ex: (p 566)

10. $x^2 - 10x = -24$

12. $z^2 + 6z - 9 = 0$

14. $y^2 + 4y = 0$

$x^2 - 10x = -24$

$x^2 - 10x + (-5)^2 = -24 + 25$

$(x-5)^2 = 1$

$x - 5 = \pm 1$

$x = 5 \pm 1$

$x = 5 + 1$ $\qquad$ $x = 5 - 1$

$x = 6$ $\qquad$ $x = 4$

$$z^2 + 6z - 9 = 0$$

$$z^2 + 6z + 3^2 = 9 + 9$$

$$(z+3)^2 = 18$$

$$z+3 = \pm\sqrt{18}$$

$$z = -3 \pm \sqrt{18}$$

$$y^2 + 4y = 0$$

$$y^2 + 4y + 2^2 = 0 + 4$$

$$(y+2)^2 = 4$$

$$y+2 = \pm 2$$

$$y+2 = 2$$

$$y = 0$$

$$y+2 = -2$$

$$y = -4$$

$$y^2 + 4y = 0$$

$$y(y+4) = 0$$

$y = 0$ $\quad y+4 = 0$

$y = -4$

18. $x^2 - 4x + 2 = 0$

$x^2 - 4x + (-2)^2 = -2 + 4$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2}$

20. $y^2 - 5y + 6 = 0$

$(y-2)(y-3) = 0$

$y-2=0 \qquad y-3=0$

$y=2 \qquad y=3$

$y^2 - 5y + 6 = 0$

$y^2 - 5y + \left(-\frac{5}{2}\right)^2 = -6 \cdot 4 + \frac{25}{4}$

$1 \cdot 4$

$\left(y - \frac{5}{2}\right)^2 = \frac{-24+25}{4}$

$\left(y - \frac{5}{2}\right)^2 = \frac{1}{4}$

$y - \frac{5}{2} = \pm \frac{1}{2}$

$y - \frac{5}{2} = \frac{1}{2}$

$y - \frac{5}{2} = -\frac{1}{2}$

$y = \frac{6}{2}$

$y = 3$

$y = \frac{4}{2}$

$y = 2$

24. $3x^2 - 12x + 14 = 0$

$$\frac{3x^2 - 12x + 14}{3} = \frac{0}{3}$$

$$x^2 - 4x + \frac{14}{3} = 0$$

$$x^2 - 4x + (-2)^2 = -\frac{14}{3} + \frac{4 \cdot 3}{1 \cdot 3}$$

$$(x-2)^2 = \frac{-14+12}{3}$$

$$(x-2)^2 = -\frac{2}{3}$$

$$x - 2 = \pm\sqrt{-\frac{2}{3}}$$

$$x = 2 \pm \sqrt{-\frac{2}{3}}$$

26. $4x^2 = -20x + 3$

$$x^2 = -5x + \frac{3}{4}$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = \frac{3}{4} + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{28}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = 7$$

$$x + \frac{5}{2} = \pm\sqrt{7}$$

$$x = -\frac{5}{2} \pm \sqrt{7}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{28}{4}$$

$$x + \frac{5}{2} = \pm\frac{\sqrt{28}}{2}$$

$$x = \frac{-5 \pm \sqrt{28}}{2}$$

9.3: Solving Quadratic Equations by the Quadratic Formula

Find the Quadratic Formula

$$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 - 5x + 6 = 0$$

$a = 1$

$b = -5$

$c = 6$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2}, \frac{5-1}{2}$$

$$x = 3, 2$$

Ex: (p 574)

8. $x^2 - 5x - 6 = 0$

10. $7k^2 + 3k - 1 = 0$

18. $5x^2 = 15$

20. $m^2 - 14 = 5m$

38. $5y^2 = 4 - y$

40. $2z^2 = z + 3$

42. $k^2 + 2k + 5 = 0$

48. $\frac{m^2}{2} = 3m - 1$

54. $\frac{2}{3}x^2 - 2x - \frac{2}{3} = 0$

58. $y^2 - 2\sqrt{5}y - 1 = 0$

9.4: Complex Solutions of Quadratic Equations

Imaginary Unit *i*

The imaginary unit, written i, is the number whose square is -1. That is,

$$i^2 = -1 \quad \text{and} \quad i = \sqrt{-1}$$

Complex Numbers and Pure Imaginary Numbers

A complex number is a number that can be written in the form

$$a + bi$$

where a and b are real numbers. A complex number that can be written in the form

$$0 + bi$$

$b \neq 0$, is also called a pure imaginary number.

Ex: (p 583)

10. $(-7 + 2i) + (5 - 3i)$

14. $(-6 + i) - (3 + i)$

18. $-2i(5 + 4i)$

20. $(6 + 2i)(4 - i)$

22. $(-9 + 2i)(-9 - 2i)$

34. $y^2 - 2y + 5 = 0$

36. $8x^2 - 7x + 2 = 0$

38. $5m^2 - 6m + 7 = 0$