

5.5: Negative Exponents & Scientific Notation

Look at division

Ex: $\frac{x^6}{x^2} = x^{6-2} = x^4$

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

$x^0 = 1, x \neq 0$

$$\frac{x^2}{x^6} = x^{2-6} = x^{-4}$$
$$\frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^4}$$

Def: $x^{-n} = \frac{1}{x^n}$

Ex:

$$x^{-3} = \frac{1}{x^3}$$

$$y^{-2} = \frac{1}{y^2}$$

$$\frac{1}{x^{-2}} = \frac{1}{\frac{1}{x^2}} = \frac{1}{1} \cdot \frac{x^2}{1} = x^2$$

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n}$$

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n$$

Ex: (p 343)

$$2. 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

$$8. \left(\frac{1}{8}\right)^{-2} = \frac{1^{-2}}{8^{-2}} = \frac{1}{8^{-2}} = 8^2 = 64$$
$$= \left(\frac{8}{1}\right)^2 = 8^2 = 64$$

$$16. \frac{r^{-5}}{s^{-2}} = \frac{\frac{1}{r^5}}{\frac{1}{s^2}} = \frac{1}{r^5} \cdot \frac{s^2}{1} = \frac{s^2}{r^5}$$

$$22. 4^{-2} - 4^{-3} =$$

$$\frac{1}{4^2} - \frac{1}{4^3} = \frac{1 \cdot 4}{16 \cdot 4} - \frac{1}{64}$$
$$= \frac{4}{64} - \frac{1}{64} = \frac{3}{64}$$

$$24. \frac{-1}{y^{-6}} = (-1) y^6 = -y^6$$

Summary of Exponent Rules

If m and n are integers and a , b , and c are real numbers, then:

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$

Power rule for exponents: $(a^m)^n = a^{m \cdot n}$

Power of a product: $(ab)^n = a^n b^n$

Power of a quotient: $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}$, $c \neq 0$

Quotient rule for exponents: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

Zero exponent: $a^0 = 1$, $a \neq 0$

Negative exponent: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

Ex: (p 343)

$$46. \frac{-5x^4y^5}{15x^4y^2} = - \frac{5^3}{3} = -\frac{1}{3}y^3$$

$$48. (-5a^4b^{-7})(-a^{-4}b^3)$$

$$+ 5a^0b^{-4} = \frac{5}{b^4}$$

$$52. \left(\frac{a^3 b}{a^7 b^{-2}} \right)^{-3} \begin{array}{l} \rightarrow (a^{-2} b^3)^{-3} \\ \downarrow \left(\frac{b^3}{a^2} \right)^{-3} \end{array}$$

$$(a^{-2} b^3)^{-3} = a^6 b^{-9} = \frac{a^6}{b^9}$$

$$\left(\frac{b^3}{a^2} \right)^{-3} = \left(\frac{a^2}{b^3} \right)^3 = \frac{a^6}{b^9}$$

$$56. \frac{5^{-1}z^7}{5^{-2}z^9} \rightarrow \frac{5^2 z^7}{5 z^9} = 5z^{-2}$$

$$= \frac{5}{z^2}$$

$$-1 - (-2)$$

$$-1 + 2$$

$$\rightarrow 5^1 z^{-2} = \frac{5}{z^2}$$

$$6^{-3} x^3 y^{-2} = \frac{x^3}{6^3 y^2}$$

$$58. \frac{6^{-5} x^{-1} y^2}{6^{-2} x^{-4} y^4} \rightarrow \frac{x^3}{6^3 y^2}$$

$$60. \left(\frac{r^{-2}s^{-3}}{r^{-4}s^{-3}} \right)^{-3} = \left(r^2 \right)^{-3} = r^{-6} = \frac{1}{r^6}$$

$$68. \frac{(a^6b^{-2})^4}{(4a^{-3}b^{-3})^3} = \frac{a^{24}b^{-8}}{64a^{-9}b^{-9}}$$

$$= \frac{a^{33}b}{64}$$

5.6: Dividing Polynomials

EX: $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7}$

Dividing a Polynomial By a Monomial

Divide each term of the polynomial by the monomial.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}, \quad c \neq 0$$

Ex: (p 350)

$$\begin{aligned} 2. \frac{15x^2 - 9x^5}{x} &= \frac{15x^2}{x} - \frac{9x^5}{x} \\ &= 15x - 9x^4 \end{aligned}$$

$$\begin{aligned} 4. \frac{8x^3 - 4x^2 + 6x + 2}{2} &= \frac{8x^3}{2} - \frac{4x^2}{2} + \frac{6x}{2} + \frac{2}{2} \\ &= 4x^3 - 2x^2 + 3x + 1 \end{aligned}$$

$$8. \frac{6x^5 + 3x^4}{3x^4} = \frac{6x^5}{3x^4} + \frac{3x^4}{3x^4} = 2x + 1$$

$$34. \frac{m^3 n^2 - mn^4}{mn} =$$

$$\frac{m^3 n^2}{mn} - \frac{mn^4}{mn} = m^2 n - n^3$$

Chapter 6: Factoring Polynomials

6.1: The Greatest Common Factor & Factoring by Grouping

Rem: find the gcf of several numbers

Factor out the gcf

To Factor a Four-Term Polynomial by Grouping

STEP 1. Group the terms in two groups of two terms so that each group has a common factor.

STEP 2. Factor out the GCF from each group.

STEP 3. If there is now a common binomial factor in the groups, factor it out.

STEP 4. If not, rearrange the terms and try these steps again.

Ex: (p 369)

26. $18a + 12$

28. $42x - 7$

30. $y^5 + 6y^4$

32. $5x^2 + 10x^6$

34. $7x + 21y - 7$

38. $x^9y^6 + x^3y^5 - x^4y^3 + x^3y^3$

40. $9y^6 - 27y^4 + 18y^2 + 6$

42. $\frac{2}{5}y^7 - \frac{4}{5}y^5 + \frac{3}{5}y^2 - \frac{2}{5}y$

44. $x(y^2 + 1) - 3(y^2 + 1)$

48. $q(b^3 - 5) + (b^3 - 5)$

50. $-7y - 21$

52. $-5y^3 + y^6$

54. $-5m^6 + 10m^5 - 5m^3$

56. $x^3 + 4x^2 + 3x + 12$

58. $xy + y + 2x + 2$

62. $8w^2 + 7wv + 8w + 7v$

64. $6x - 42 + xy - 7y$

70. $6m^2 - 5mn - 6m + 5n$

74. $90 + 15y^2 - 18x - 3xy^2$

86. $16x^2 + 4xy^2 + 8xy + 2y^3$

6.2: Factoring Trinomials of the Form $x^2 + bx + c$

Multiply out two binomials

Factoring a Trinomial of the Form $x^2 + bx + c$

The factored form of $x^2 + bx + c$ is

The product of these numbers is c .

$$x^2 + bx + c = (x + \square)(x + \square)$$

The sum of these numbers is b .

Ex: (p 376)

2. $x^2 + 6x + 8$

4. $y^2 - 12y + 11$

6. $x^2 - 10x + 25$

8. $x^2 - x - 30$

10. $x^2 + 4x - 32$

12. $x^2 - 7x + 5$

48. $3x^3 + 3x^2 - 126x$

50. $3x^2y - 9xy + 45y$

54. $x^2 - 3xy - 4y^2$

62. $7a^3b - 35a^2b^2 + 42ab^3$

66. $-x^2 + 8x - 7$

68. $\frac{1}{3}y^2 - \frac{5}{3}y - 8$

6.3: Factoring Trinomials of the Form $ax^2 + bx + c$ and Perfect Square Trinomials

Multiply two binomials

Two Strategies: guess & check
key numbering (by grouping)

Ex: (p 384)

2. $2y^2 + 27y + 25 = (2y + 25)(\quad)$

4. $6y^2 + 11y - 10 = (2y + 5)(\quad)$

6. $4y^2 - 20y + 25 = (2y - 5)(\quad)$

8. $3x^2 + 8x + 4$

10. $21x^2 - 31x + 10$

14. $3x^2 + 20x - 63$

18. $3n^2 + 20n + 5$

20. $8x^2 - 14xy + 3y^2$

24. $8a^3 + 14a^2 + 3a$

30. $8x^2y + 34xy - 84y$

34. $-x^2 + 4x + 21$

40. $x^2 + 18x + 81$

42. $x^2 - 12x + 36$

44. $25x^2 - 20x + 4$

46. $m^4 + 10m^2 + 25$

48. $3y^2 - 6y + 3$

50. $9y^2 + 48y + 64$

52. $2x^2 + 7x - 72$

57. $-9x + 20 + x^2$

60. $m^2 + 20mn + 100n^2$

72. $-15x^2 + 26x - 8$

68. $12x^3 - 34x^2 + 24x$

74. $9q^4 - 42q^3 + 49q^2$

80. $1 + 16x^2 + x^4$

92. $3a^2b^2 + 12ab + 1$

Ex: (p 390)

14. $15x^2 + 11x + 2$

20. $2x^2 - 7x + 3$

34. $30a^2 + 38a - 20$

6.5: Factoring Binomials

Ex: (p 396)

2. $x^2 - 36$

6. $49a^2 - 16$

14. $-9t^2 + 1$

20. $n^4 - 16$

38. $x^2 - 225y^2$

44. $36x^2y - 25y$

56. $100 - \frac{4}{81}n^2$

64. $100x^3y - 49xy^3$

70. $25y^4 - 100y^2$

6.6: Solving Quadratic Equations by Factoring

Quadratic Equation

A quadratic equation is one that can be written in the form

$$ax^2 + bx + c = 0$$

where $a, b,$ and c are real numbers and $a \neq 0$.

Zero Factor Theorem

If a and b are real numbers and if $ab = 0$, then $a = 0$ or $b = 0$.

Ex: (p 408)

2. $(x + 4)(x - 10) = 0$

4. $(x + 11)(x + 1) = 0$

6. $x(x - 7) = 0$

20. $x^2 + 2x - 63 = 0$

22. $x^2 - 5x + 6 = 0$

24. $x^2 - 3x = 0$

28. $x^2 = 9$

30. $(x + 3)(x + 8) = x$

32. $x(4x - 11) = 3$

34. $-2y^2 + 72 = 0$

36. $6x^2 + 57x = 30$

42. $4y^3 - 36y = 0$

44. $15x^3 + 24x^2 - 63x = 0$

46. $(x - 6)(x + 7) = 0$

48. $x^2 + 15x = 0$

50. $5(3 - 4x) = 9$

52. $4y^2 - 81 = 0$

60. $9x^2 + 7x = 2$

62. $3x^2 - 6x - 9 = 0$

64. $(y - 5)(y - 2) = 28$

74. $2x^2 + 12x - 1 = 4 + 3x$

76. $4x^2 - 20x = -5x^2 - 6x - 5$

