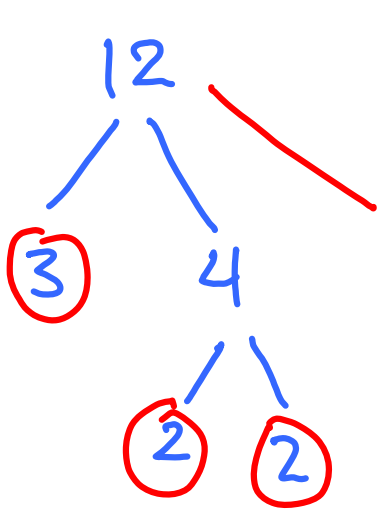
Chapter 6: Factoring Polynomials

Ex: Numbers - Factors

12 : 1, 12, 2, 6, 3, 4



prime factors

$2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

6.1: The Greatest Common Factor & Factoring by Grouping

Rem: find the gcf of several numbers

Factor out the gcf

Ex: $9x + 12xy$

$3x(3 + 4y)$

$2x^2y^3 + 6x^4y^7 - 8x^2y^5$

$2x^2y^3(1 + 3x^2y^4 - 4y^2)$

$16x^6y^5 - 12x^3y^7 + 8x^4y^3 - 10x^7y^{10}$

$2x^3y^3(8x^3y^2 - 6y^4 + 4x - 5x^4y^7)$

To Factor a Four-Term Polynomial by Grouping

STEP 1. Group the terms in two groups of two terms so that each group has a common factor.

STEP 2. Factor out the GCF from each group.

STEP 3. If there is now a common binomial factor in the groups, factor it out.

STEP 4. If not, rearrange the terms and try these steps again.

Ex: (p 369)

26. $18a + 12$ $= 6(3a+2)$

28. $42x - 7$ $= 7(6x-1)$

30. $y^5 + 6y^4$ $= y^4(y+6)$

32. $5x^2 + 10x^6$ $= 5x^2(1+2x^4)$

34. $7x + 21y - 7$ $= 7(x+3y-1)$

38. $x^9y^6 + x^3y^5 - x^4y^3 + x^3y^3$

$x^3y^3(x^6y^3 + y^2 - x + 1)$

40. $9y^6 - 27y^4 + 18y^2 + 6$

42. $\frac{2}{5}y^7 - \frac{4}{5}y^5 + \frac{3}{5}y^2 - \frac{2}{5}y$

$\frac{1}{5}y(2y^6 - 4y^4 + 3y - 2)$

$xa - 3a = a(x-3)$

44. $x(y^2 + 1) - 3(y^2 + 1)$

$(y^2+1)(x-3)$

$q(a) + a$

$a(q+1)$

48. $q(b^3 - 5) + (b^3 - 5)$

$(b^3-5)(q+1)$

50. $-7y - 21$

52. $-5y^3 + y^6$

54. $-5m^6 + 10m^5 - 5m^3$

56. $x^3 + 4x^2 + 3x + 12$

$x^2(x+4) + 3(x+4)$

$(x+4)(x^2+3)$

58. $xy + y + 2x + 2$

$y(x+1) + 2(x+1)$

$(x+1)(y+2)$

62. $8w^2 + 7wv + 8w + 7v$

$w(8w+7v) + 1(8w+7v)$

$(8w+7v)(w+1)$

64. $6x - 42 + xy - 7y$

$6(x-7) + y(x-7)$

$(x-7)(6+y)$

70. $6m^2 - 5mn - 6m + 5n$

$m(6m - 5n) - 1(6m - 5n)$

$(6m - 5n)(m - 1)$

74. $90 + 15y^2 - 18x - 3xy^2$

$3(30 + 5y^2 - 6x - xy^2)$

$3[5(6 + y^2) - x(6 + y^2)]$

$3(6 + y^2)(5 - x)$

86. $16x^2 + 4xy^2 + 8xy + 2y^3$

$2[8x^2 + 2xy^2 + 4xy + y^3]$

$2[2x(4x + y^2) + y(4x + y^2)]$

$2(4x + y^2)(2x + y)$

6.2: Factoring Trinomials of the Form $x^2 + bx + c$

Multiply out two binomials

Factoring a Trinomial of the Form $x^2 + bx + c$

The factored form of $x^2 + bx + c$ is

The product of these numbers is c.

$$x^2 + bx + c = (x + \square)(x + \square)$$

The sum of these numbers is b.

Ex: (p 376)

2. $x^2 + 6x + 8$

4. $y^2 - 12y + 11$

6. $x^2 - 10x + 25$

8. $x^2 - x - 30$

10. $x^2 + 4x - 32$

12. $x^2 - 7x + 5$

48. $3x^3 + 3x^2 - 126x$

50. $3x^2y - 9xy + 45y$

54. $x^2 - 3xy - 4y^2$

62. $7a^3b - 35a^2b^2 + 42ab^3$

66. $-x^2 + 8x - 7$

68. $\frac{1}{3}y^2 - \frac{5}{3}y - 8$

6.3: Factoring Trinomials of the Form $ax^2 + bx + c$ and Perfect Square Trinomials

Multiply two binomials

Two Strategies: guess & check
key numbering (by grouping)

Ex: (p 384)

2. $2y^2 + 27y + 25 = (2y + 25)(\quad)$

4. $6y^2 + 11y - 10 = (2y + 5)(\quad)$

6. $4y^2 - 20y + 25 = (2y - 5)(\quad)$

8. $3x^2 + 8x + 4$

10. $21x^2 - 31x + 10$

14. $3x^2 + 20x - 63$

18. $3n^2 + 20n + 5$

20. $8x^2 - 14xy + 3y^2$

24. $8a^3 + 14a^2 + 3a$

30. $8x^2y + 34xy - 84y$

34. $-x^2 + 4x + 21$

40. $x^2 + 18x + 81$

42. $x^2 - 12x + 36$

44. $25x^2 - 20x + 4$

46. $m^4 + 10m^2 + 25$

48. $3y^2 - 6y + 3$

50. $9y^2 + 48y + 64$

52. $2x^2 + 7x - 72$

57. $-9x + 20 + x^2$

60. $m^2 + 20mn + 100n^2$

72. $-15x^2 + 26x - 8$

68. $12x^3 - 34x^2 + 24x$

74. $9q^4 - 42q^3 + 49q^2$

80. $1 + 16x^2 + x^4$

92. $3a^2b^2 + 12ab + 1$

Ex: (p 390)

14. $15x^2 + 11x + 2$

20. $2x^2 - 7x + 3$

34. $30a^2 + 38a - 20$

6.5: Factoring Binomials

Ex: (p 396)

2. $x^2 - 36$

6. $49a^2 - 16$

14. $-9t^2 + 1$

20. $n^4 - 16$

38. $x^2 - 225y^2$

44. $36x^2y - 25y$

56. $100 - \frac{4}{81}n^2$

64. $100x^3y - 49xy^3$

70. $25y^4 - 100y^2$

6.6: Solving Quadratic Equations by Factoring

Quadratic Equation

A quadratic equation is one that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

Zero Factor Theorem

If a and b are real numbers and if $ab = 0$, then $a = 0$ or $b = 0$.

Ex: (p 408)

2. $(x + 4)(x - 10) = 0$

4. $(x + 11)(x + 1) = 0$

6. $x(x - 7) = 0$

20. $x^2 + 2x - 63 = 0$

22. $x^2 - 5x + 6 = 0$

24. $x^2 - 3x = 0$

28. $x^2 = 9$

30. $(x + 3)(x + 8) = x$

32. $x(4x - 11) = 3$

34. $-2y^2 + 72 = 0$

36. $6x^2 + 57x = 30$

42. $4y^3 - 36y = 0$

44. $15x^3 + 24x^2 - 63x = 0$

46. $(x - 6)(x + 7) = 0$

48. $x^2 + 15x = 0$

50. $5(3 - 4x) = 9$

52. $4y^2 - 81 = 0$

60. $9x^2 + 7x = 2$

62. $3x^2 - 6x - 9 = 0$

64. $(y - 5)(y - 2) = 28$

74. $2x^2 + 12x - 1 = 4 + 3x$

76. $4x^2 - 20x = -5x^2 - 6x - 5$