$x^2 \cdot x^3 = (xx)(xxx) = x^5$

Product Rule for Exponents

If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}$$

← Add exponents.

↑ Keep common base.

$w^2 \cdot w^7 = w^9$

$x^2 \cdot x^1 = x^3$

$x^3 y^2 = x^3 y^2$

$2^2 \cdot 2^3 = 2^5 = 32$

$(x^2)^3 = (x^2)(x^2)(x^2) = x^6$

Power Rule for Exponents

If m and n are positive integers and a is a real number, then

$$(a^m)^n = a^{mn}$$

mn ← Multiply exponents.

a ← Keep common base.

$(x^3)^4 = x^{12}$

$(y)^6 = y^6$

$(2^3)^7 = 2^{21}$

$(-3)^2 = 9$ $\qquad -3^2 = -9$

Power of a Quotient Rule

If n is a positive integer and a and c are real numbers, then

$$\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}, \quad c \neq 0$$

$$\left(\frac{x^2}{y^3}\right)^7 = \frac{x^{14}}{y^{21}}$$

$$(x^2y^3)^7 = x^{14}y^{21}$$

$$(xy)^2 = x^2y^2$$

$$(x+y)^2 = \text{TROUBLE!}$$

$$\frac{x^6}{x^2} = \frac{\cancel{x}\cancel{x}xxxx}{\cancel{x}\cancel{x}} = \frac{x^4}{1} = x^4$$

Quotient Rule for Exponents

If m and n are positive integers and a is a real number, then

$$\frac{a^m}{a^n} = a^{m-n}$$

as long as a is not 0.

$$\frac{x^6}{x^3} = x^3$$

$$\frac{y^{10}}{y^4} = y^6$$

$$\frac{y^6}{x^2} = \frac{y^6}{x^2}$$

$$\frac{x^4}{x^4} = 1 \qquad x^{4-4} = x^0$$

Zero Exponent

$a^0 = 1$, as long as a is not 0.

$$x^0 = 1, \quad x \neq 0$$

Prelim: $\frac{a}{b} = c$ is the same as $a = bc$

$\frac{8}{2} = 4$ $\qquad 8 = 2(4)$

Aside

$\frac{0}{0} = $ ~~x~~ undefined $\qquad 0 = 0 \cdot 5 = 0$

$\frac{0}{0} = 3 \qquad 0 = 0 \cdot 3 = 0$

Ex: (p 310)

2. $-3^2 = (-1)3^2 = -9$

4. $(-3)^2 = (-3)(-3) = 9$

6. $-4^3 = (-1)4^3 = -64$

8. $(-4)^3 = (-4)(-4)(-4) = -64$

28. $y^2 \cdot y = y^3$

30. $(-5)^7 \cdot (-5)^6 = -5^{13}$

32. $(-2z^3)(-2z^2) = 4z^5$

34. $(a^2b)(a^{13}b^{17}) = a^{15}b^{18}$

36. $(-7a^3b^3)(7a^{19}b) = -49a^{22}b^4$

38. $(12x^5)(-x^6)(x^4) = -12x^{15}$

32. $(-2z^3)(-2z^2)$ $(-2)^2 = 4$

$4z^5$

44. $(ab)^6 = a^6b^6$

46. $(4x^6)^2 = 16x^{12}$

48. $(a^4b)^7 = a^{28}b^7$

50. $(-3x^7yz^2)^3 = -27x^{21}y^3z^6$

52. $\left(\frac{q}{t}\right)^{11} = \frac{q^{11}}{t^{11}}$

54. $\left(\frac{xy}{7}\right)^2 = \frac{x^2y^2}{49}$

56. $\left(\frac{xy^4}{-3z^3}\right)^3 = \frac{x^3y^{12}}{-27z^9}$

62. $\dfrac{y^{10}}{y^{9}}$

64. $\dfrac{(-6)^{13}}{(-6)^{11}}$

66. $\dfrac{x^{8}y^{6}}{xy^{5}}$

68. $\dfrac{9a^{4}b^{7}}{27ab^{2}}$

70. 23^0

72. $(4y)^0$

74. $-2x^0$

76. $-3^0 + 4^0$

78. $(-9)^2$

80. $\left(\frac{2}{3}\right)^3$

82. $\left(\frac{pt}{3}\right)^3$

86. $(3y^4)(-5y)$

88. $(y^2z^2)(y^{15}z^{13})$

90. $(-3s^5t)(-7st^{10})$

5.2: Adding and Subtracting Polynomials

Expression, term

Coefficient

Polynomial

A **polynomial in x** is a finite sum of terms of the form ax^n, where a is a real number and n is a whole number.

Types of Polynomials

A **monomial** is a polynomial with exactly one term.

A **binomial** is a polynomial with exactly two terms.

A **trinomial** is a polynomial with exactly three terms.

Degree of a Term

The degree of a term is the sum of the exponents on the variables contained in the term.

Ex: (p 320)

Find the degree of a polynomial

Find the value of each polynomial when **(a)** $x = 0$ *and* **(b)** $x = -1$. *See Examples 4 and 5.*

16. $x^2 - 4$

18. $-2x^3 + 3x^2 - 6$

Simplify

24. $18x^3 - 4x^3$

32. $\frac{1}{6}x^4 - \frac{1}{7}x^2 + 5 - \frac{1}{2}x^4 - \frac{3}{7}x^2 + \frac{1}{3}$

Perform the Following:

36. $(3x^2 + 7) + (3x^2 + 9)$

40. $(5x^2 + 4) - (-2y^2 + 4)$

44. $(-7x^2 + 4x + 7) - (-8x + 2)$

46.
$$\begin{array}{r} 7x^3 + 3 \\ +2x^3 + 1 \\ \hline \end{array}$$

48.
$$\begin{array}{r} 5u^5 - 4u^2 + 3u - 7 \\ -(3u^5 + 6u^2 - 8u + 2) \\ \hline \end{array}$$

60. $(6y^5 - 6y^3 + 4) + (-2y^5 - 8y^3 - 7)$

62. $(-a^2 + 1) - (a^2 - 3) + (5a^2 - 6a + 7)$

68. Subtract $(5y + 7x^2)$ from the sum of $(8y - x)$ and $(3 + 8x^2)$.

70. Subtract $(4x^2 - 2x + 2)$ from the sum of $(x^2 + 7x + 1)$ and $(7x + 5)$.

80. $(3x - 2 + 6y) + (7x - 2 - y)$

86. $(3x^2y - 6xy + x^2y^2 - 5) - (11x^2y^2 - 1 + 5yx^2)$

92. $-7x(x)$

94. $6r^3(7r^{10})$

96. $-z^2y(11zy)$

5.3: Multiplying Polynomials

Ex: (p 327)

2. $9t^6(-3t^5)$

4. $(-5.2x^4)(3x^4)$

6. $\left(-\frac{3}{4}y^7\right)\left(\frac{1}{7}y^4\right)$

8. $(x)(5x^4)(-6x^7)$

18. $-x(6y^3 - 5xy^2 + x^2y - 5x^3)$

22. $(x + 2)(x + 9)$

24. $(y - 10)(y + 11)$

26. $\left(x + \frac{3}{5}\right)\left(x - \frac{2}{5}\right)$

28. $(5x^2 + 2)(6x^2 + 2)$

34. $(x^2 + 4)^2$

38. $(x + 3)(x^2 + 5x - 8)$

42. $(3 + b)(2 - 5b - 3b^2)$

44. $(y - 1)^3$

46. $(3x + 4)^3$

50. $(4x - 5)(8x^2 + 2x - 4)$

52. $(3x^2 - x + 2)(x^2 + 2x + 1)$

56. $-5x(x^2 - 3x + 10)$

60. $\left(m + \frac{2}{9}\right)\left(m - \frac{1}{9}\right)$

68. $(5x + 4)(x^2 - x + 4)$

70. $(a^2 + 3a - 2)(2a^2 - 5a - 1)$

5.4: Special Products

Ex: (p 334)

4. $(y - 12)(y + 4)$

6. $(3y - 5)(2y - 7)$

12. $(x + 7)^2$

24. $(4x - 5)(4x + 5)$

26. $\left(10x + \frac{2}{7}\right)\left(10x - \frac{2}{7}\right)$

28. $(2x - y)(2x + y)$

36. $(6a + 7)(6a + 5)$

38. $(x - 10)(x + 10)$

42. $(x^3 - 2)(5x + y)$

44. $(x - 2)(x^2 - 4x + 2)$

48. $(11x - 7y)(11x + 7y)$

54. $(x^5 + 5)(x^2 - 8)$

62. $\left(\frac{2}{3}a - b^2\right)\left(\frac{2}{3}a - b^2\right)$

70. $(3x + 5)(3x - 5)$

74. $\left(\frac{a}{2} + 4y\right)\left(\frac{a}{2} - 4y\right)$

78. $(b + 3)(2b^2 + b - 3)$

82. $\frac{x^3y^6}{xy^2}$

84. $\frac{-6a^8y}{3a^4y}$

86. $\frac{-48ab^6}{32ab^3}$

5.5: Negative Exponents & Scientific Notation

Look at division

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n}$$

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n$$

Ex: (p 343)

2. 6^{-2}

8. $\left(\frac{1}{8}\right)^{-2}$

16. $\frac{r^{-5}}{s^{-2}}$

22. $4^{-2} - 4^{-3}$

24. $\frac{-1}{y^{-6}}$

Summary of Exponent Rules

If m and n are integers and a, b, and c are real numbers, then:

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$

Power rule for exponents: $(a^m)^n = a^{m \cdot n}$

Power of a product: $(ab)^n = a^n b^n$

Power of a quotient: $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}, \quad c \neq 0$

Quotient rule for exponents: $\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$

Zero exponent: $a^0 = 1, \quad a \neq 0$

Negative exponent: $a^{-n} = \frac{1}{a^n}, \quad a \neq 0$

Ex: (p 343)

46. $\dfrac{-5x^4y^5}{15x^4y^2}$

48. $(-5a^4b^{-7})(-a^{-4}b^3)$

52. $\left(\dfrac{a^5b}{a^7b^{-2}}\right)^{-3}$

56. $\dfrac{5^{-1}z^7}{5^{-2}z^9}$

58. $\dfrac{6^{-5}x^{-1}y^2}{6^{-2}x^{-4}y^4}$

60. $\left(\dfrac{r^{-2}s^{-3}}{r^{-4}s^{-3}}\right)^{-3}$

68. $\dfrac{(a^6b^{-2})^4}{(4a^{-3}b^{-3})^3}$

5.6: Dividing Polynomials

Dividing a Polynomial By a Monomial

Divide each term of the polynomial by the monomial.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}, \quad c \neq 0$$

Ex: (p 350)

2. $\dfrac{15x^2 - 9x^5}{x}$

4. $\dfrac{8x^3 - 4x^2 + 6x + 2}{2}$

8. $\dfrac{6x^5 + 3x^4}{3x^4}$

34. $\dfrac{m^3n^2 - mn^4}{mn}$