

Table GFL -1. Dimensionless eigenvalues for large cotime Green's functions.

Number	Eigenvalue, $\xi_m$	Eigencondition
X11	$mB, m = 1, 2, \dots$	
X12	$(m-1/2)B, m = 1, 2, \dots$	
X13		$\xi_m \cot(\xi_m) = -Bi_{XL} = -h_{xL}L/k$
X21	$(m-1/2)B, m = 1, 2, \dots$	
X22	$\xi_0 = 0$	
	$\xi_m = mB, m = 1, 2, \dots$	
X23		$\xi_m \tan(\xi_m) = Bi_{XL} = h_{xL}L/k$
X31		$\xi_m \cot(\xi_m) = -Bi_{X0} = -h_{x0}L/k$
X32		$\xi_m \tan(\xi_m) = Bi_{X0} = h_{x0}L/k$
X33		$\tan(\xi_m L) = M$

$$\Phi = \frac{\beta_m (Bi_{X0} + Bi_{XL})}{\beta_m^2 - Bi_{X0} Bi_{XL}}$$

Table GFL -2A. Eigenfunctions for subroutine EIGFUN. Dimensionless  $x^+$  and eigenvalues. Large time Green's functions.

Number	Eigenfunction
X11	$E_{X11}(x^+, \mathcal{S}_m) = \sin(\mathcal{S}_m x^+)$
X12	$E_{X12}(x^+, \mathcal{S}_m) = \sin(\mathcal{S}_m x^+)$
X13	$E_{X13}(x^+, \mathcal{S}_m) = \sin(\mathcal{S}_m x^+)$
X21	$E_{X21}(x^+, \mathcal{S}_m) = \cos(\mathcal{S}_m x^+)$
X22	$E_{X22}(x^+, \mathcal{S}_m) = 1$ for $m = 0$ $E_{X22}(x^+, \mathcal{S}_m) = \cos(\mathcal{S}_m x^+)$ , $m = 1, 2, \dots$
X23	$E_{X23}(x^+, \mathcal{S}_m) = \cos(\mathcal{S}_m x^+)$
X31	$E_{X31}(x^+, \mathcal{S}_m) = \sin[\mathcal{S}_m(1-x^+)]$
X32	$E_{X32}(x^+, \mathcal{S}_m) = \cos[\mathcal{S}_m(1-x^+)]$
X33	$E_{X33}(x^+, \mathcal{S}_m) = [\mathcal{S}_m \cos(\mathcal{S}_m x^+) + Bi_{X0} \sin(\mathcal{S}_m x^+)] / [\mathcal{S}_m^2 + Bi_{X0}^2]^{1/2}$

All these eigenfunctions have values between positive and negative one.

Table GFL -2B. Derivatives for eigenfunctions for subroutine EIGFUN. Dimensionless  $x^+$  and eigenvalues. Large time Green's functions,  $x^+$ .

Number	Derivative of eigenfunction
X11	$\mathcal{M}E_{X11}(x^+, \mathcal{S}_m) / \mathcal{M} = \cos(\mathcal{S}_m x^+) \mathcal{S}_m / L$
X12	$\mathcal{M}E_{X12}(x^+, \mathcal{S}_m) / \mathcal{M} = \cos(\mathcal{S}_m x^+) \mathcal{S}_m / L$
X13	$\mathcal{M}E_{X13}(x^+, \mathcal{S}_m) / \mathcal{M} = \cos(\mathcal{S}_m x^+) \mathcal{S}_m / L$
X21	$\mathcal{M}E_{X21}(x^+, \mathcal{S}_m) / \mathcal{M} = -\sin(\mathcal{S}_m x^+) \mathcal{S}_m / L$
X22	$\mathcal{M}E_{X22}(x^+, \mathcal{S}_m) / \mathcal{M} = 0$ for $m = 0$ $\mathcal{M}E_{X22}(x^+, \mathcal{S}_m) / \mathcal{M} = -\sin(\mathcal{S}_m x^+) \mathcal{S}_m / L$ , $m = 1, 2, \dots$
X23	$\mathcal{M}E_{X23}(x^+, \mathcal{S}_m) / \mathcal{M} = -\sin(\mathcal{S}_m x^+) \mathcal{S}_m / L$
X31	$\mathcal{M}E_{X31}(x^+, \mathcal{S}_m) / \mathcal{M} = -\cos[\mathcal{S}_m(1-x^+)] \mathcal{S}_m / L$
X32	$\mathcal{M}E_{X32}(x^+, \mathcal{S}_m) / \mathcal{M} = \sin[\mathcal{S}_m(1-x^+)] \mathcal{S}_m / L$
X33	$\mathcal{M}E_{X33}(x^+, \mathcal{S}_m) / \mathcal{M} = [-\mathcal{S}_m \sin(\mathcal{S}_m x^+) + Bi_{X0} \cos(\mathcal{S}_m x^+)] (\mathcal{S}_m / L) / [\mathcal{S}_m^2 + Bi_{X0}^2]^{1/2}$

Table GFL - 3. Large cotime eigenfunctions evaluated at a boundary. Dimensionless  $x$  and eigenvalues. For boundary conditions of the first kind at the evaluated boundary,  $-M/MN$  is evaluated at the boundary.

Number	At $x^+N= 0$	At $x^+N= 1$
X11	$-M_{X11}(0, \xi_m)/M^+N= mB$	$-M_{X11}(L, \xi_m)/M^+N= mB(-1)^{m+1}$
X12	$-M_{X12}(0, \xi_m)/M^+N= (2m-1)B/2$	$E_{X12}(L, \xi_m) = (-1)^{m+1}$
X13	$-M_{X13}(0, \xi_m)/M^+N= \xi_m$	$E_{X13}(L, \xi_m) = \sin(\xi_m)$
X21	$E_{X21}(0, \xi_m) = 1$	$-M_{X21}(L, \xi_m)/M^+N= \xi_m (-1)^{m+1}$
X22	$E_{X22}(0, \xi_m) = 1$ for $m = 0$ $E_{X22}(0, \xi_m) = 1$ for $m = 1, 2, ..$	$E_{X22}(L, \xi_m) = 1$ for $m = 0$ $E_{X22}(L, \xi_m) = (-1)^m$ for $m = 1, 2, ..$
X23	$E_{X23}(0, \xi_m) = 1$	$E_{X23}(L, \xi_m) = \cos(\xi_m)$
X31	$E_{X31}(0, \xi_m) = \sin(\xi_m)$	$-M_{X31}(L, \xi_m)/M^+N= \xi_m$
X32	$E_{X32}(0, \xi_m) = \cos(\xi_m)$	$E_{X32}(L, \xi_m) = 1$
X33	$E_{X33}(0, \xi_m) = \xi_m / [\xi_m^2 + Bi_{X0}^2]^{1/2}$	$E_{X33}(L, \xi_m) = [\xi_m \cos(\xi_m) + Bi_{X0} \sin(\xi_m)] / [\xi_m^2 + Bi_{X0}^2]^{1/2}$

Table GFL - 4. Dimensionless norms for large cotime Green's functions.

Number	Norm, $N_{XIJ}$
X11	$N_{X11} = 1/2$
X12	$N_{X12} = 1/2$
X13	$N_{X13} = \mathcal{N}_{XL}/2$
	$\varphi_{XL} = 1 + \frac{Bi_{XL}}{\beta_m^2 + Bi_{XL}^2}$
X21	$N_{X21} = 1/2$
X22	$N_{X22} = 1$ for $m = 0$ $N_{X22} = 1/2$ for $m = 1, 2, \dots$
X23	$N_{X23} = \mathcal{N}_{XL}/2$
X31	$N_{X31} = \mathcal{N}_{X0}/2$
	$\varphi_{XL} = 1 + \frac{Bi_{X0}}{\beta_m^2 + Bi_{X0}^2}$
X32	$N_{X32} = \mathcal{N}_{X0}/2$
X33	$N_{X33} = \mathcal{N}_{X0L}/2$
	$\varphi_{X0L} = 1 + \frac{Bi_{X0}}{\beta_m^2 + Bi_{X0}^2} + \frac{Bi_{XL}}{\beta_m^2 + Bi_{XL}^2}$

Each of these dimensionless norms is equal to or greater than 1/2. As  $m$  goes to infinity, each dimensionless norm goes to 1/2.