

Table 1. Short-time Green's functions. The G_{X00} , G_{X10} , G_{X20} and G_{X30} cases are exact. The other cases are not exact but are very accurate. Expressions for the y -direction are found by replacing x by y , $x \setminus \mathbb{N}$ by $y \setminus \mathbb{N}$ and L by W . Similar results are obtained for the z -direction by using z , $z \setminus \mathbb{N}$ and H .

| Number | Equation |
|---------|---|
| 1A.X00 | $G_{X10}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N})$ |
| 1A.X10 | $G_{X10}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) - K(x + x \setminus \mathbb{N})$ |
| 1A.X11 | $G_{X11}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) - K(x + x \setminus \mathbb{N}) - K(2L - x - x \setminus \mathbb{N}) + K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N})$ |
| 1A.X12 | $G_{X12}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) - K(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) - K(2L - x + x \setminus \mathbb{N}) - K(2L + x - x \setminus \mathbb{N})$ |
| 1A.X13 | $G_{X13}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) - K(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) - K(2L - x + x \setminus \mathbb{N}) - K(2L + x - x \setminus \mathbb{N}) - (h_L/k)H_L(2L - x - x \setminus \mathbb{N}) + (h_L/k)H_L(2L - x + x \setminus \mathbb{N}) + (h_L/k)H_L(2L + x - x \setminus \mathbb{N})$ |
| 1A.X20 | $G_{X20}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N})$ |
| 1A.X21 | $G_{X21}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) - K(2L - x - x \setminus \mathbb{N}) - K(2L - x + x \setminus \mathbb{N}) - K(2L + x - x \setminus \mathbb{N})$ |
| 1A.X22 | $G_{X22}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) + K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N})$ |
| 1A.X23 | $G_{X23}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) - (h_L/k)H_L(2L - x - x \setminus \mathbb{N}) + K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N}) - (h_L/k)H_L(2L - x + x \setminus \mathbb{N}) - (h_L/k)H_L(2L + x - x \setminus \mathbb{N})$ |
| 1A.X30 | $G_{X30}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) - (h_0/k)H_0(x + x \setminus \mathbb{N})$ |
| 1A.X31 | $G_{X31}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) - (h_0/k)H_0(x + x \setminus \mathbb{N}) - K(2L - x - x \setminus \mathbb{N}) - K(2L - x + x \setminus \mathbb{N}) - K(2L + x - x \setminus \mathbb{N}) + (h_0/k)H_0(2L - x + x \setminus \mathbb{N}) + (h_0/k)H_0(2L + x - x \setminus \mathbb{N})$ |
| 1A.X32 | $G_{X32}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) - (h_0/k)H_0(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) + K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N}) - (h_0/k)H_0(2L - x + x \setminus \mathbb{N}) - (h_0/k)H_0(2L + x - x \setminus \mathbb{N})$ |
| 1A.X33a | $G_{X33}(x, x \setminus \mathbb{N} u) = K(x - x \setminus \mathbb{N}) + K(x + x \setminus \mathbb{N}) - (h_0/k)H_0(x + x \setminus \mathbb{N}) + K(2L - x - x \setminus \mathbb{N}) - (h_L/k)H_L(2L - x - x \setminus \mathbb{N}) + K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N}) - (h_0/k)H_0(2L - x + x \setminus \mathbb{N}) - (h_0/k)H_0(2L + x - x \setminus \mathbb{N}) - (h_L/k)H_L(2L - x + x \setminus \mathbb{N}) - (h_L/k)H_L(2L + x - x \setminus \mathbb{N}) + JH(L, x, x \setminus \mathbb{N})$ |
| | where for $h_0 \dots h_L$ |
| 1A.X33b | $JH(L, x, x \setminus \mathbb{N}) = (2h_0 h_L / (k(h_L - h_0))) [H_0(2L - x + x \setminus \mathbb{N}) + H_0(2L + x - x \setminus \mathbb{N}) - H_L(2L - x + x \setminus \mathbb{N}) - H_L(2L + x - x \setminus \mathbb{N})]$ |
| | and for $h_0 = h_L$ |
| 1A.X33c | $JH(L, x, x \setminus \mathbb{N}) = (2h_0^2 / k^2) [4 "u [K(2L - x + x \setminus \mathbb{N}) + K(2L + x - x \setminus \mathbb{N})] - (2L - x + x \setminus \mathbb{N}) +$ |

$$2(h_0/k) {}^u H_0(2L-x+x) - (2L+x-x) + 2(h_0/k) {}^u H_0(2L+x-x)$$

Some Limiting Relations

- As h_i goes to zero, $H_i(z)$ goes to $E(z)$. As h_i goes to infinity, $H_i(z)$ goes to 0.
- As h_i goes to zero, $(h_i/k) H_i(z)$ goes to 0. As h_i goes to infinity, $(h_i/k) H_i(z)$ goes to $2K(z)$.
- As h_i goes to infinity, $(h_i/k)[(h_i/k)H_i(z) - 2K(z)]$ goes to $-2zK(z)/(2 {}^u)$.
- As h_0 goes to zero, $JH(L,x,x')$ goes to 0. As h_0 goes to infinity, $JH(L,x,x')$ goes to $2(h_l/k)[H_l(2L-x+x') + H_l(2L+x-x')]$.
- As h_l goes to zero, $JH(L,x,x')$ goes to 0. As h_l goes to infinity, $JH(L,x,x')$ goes to $2(h_0/k)[H_0(2L-x+x') + H_0(2L+x-x')]$.
- As u goes to 0 for $x \dots 0$, $E(z)$ goes to 0 and $H_i(z)$ goes to 0.

Table 2A. Short-time Green's functions for boundary conditions of the 2nd and 3rd kinds and negative of the derivative of the Green's function (with respect to the outward pointing normal) for the 1st kind of boundary condition. Evaluated at $x_N = 0$.

| Number | Equation |
|---------|--|
| 2A.X10 | $-\mathcal{M}G_{X10}(x,0,u)/\mathcal{M}N = (u)^{-1} xK(x)$ |
| 2A.X11 | $-\mathcal{M}G_{X11}(x,0,u)/\mathcal{M}N = (u)^{-1} [xK(x) - (2L-x)K(2L-x) + (2L+x)K(2L+x)], x \dots 0$ |
| 2A.X12 | $-\mathcal{M}G_{X12}(x,0,u)/\mathcal{M}N = (u)^{-1} [xK(x) + (2L-x)K(2L-x) - (2L+x)K(2L+x)], x \dots 0$ |
| 2A.X13 | $-\mathcal{M}G_{X13}(x,0,u)/\mathcal{M}N = (u)^{-1} [xK(x) + (2L-x)K(2L-x) - (2L+x)K(2L+x) + 2(h_L/k)^2 H_L(2L-x) - (4h_L/k)K(2L-x) - (h_L/k)^2 H_L(2L+x) + (2h_L/k)K(2L+x)], x \dots 0$ |
| 2A.X20 | $G_{X20}(x,0,u) = 2K(x)$ |
| 2A.X21 | $G_{X21}(x,0,u) = 2[K(x) - K(2L-x) - K(2L+x)]$ |
| 2A.X22 | $G_{X22}(x,0,u) = 2[K(x) + K(2L-x) + K(2L+x)]$ |
| 2A.X23 | $G_{X23}(x,0,u) = 2[K(x) + K(2L-x) + K(2L+x)] - 2(h_L/k)H_L(2L-x) - (h_L/k)H_L(2L+x)$ |
| 2A.X30 | $G_{X30}(x,0,u) = 2K(x) - (h_0/k)H_0(x)$ |
| 2A.X31 | $G_{X31}(x,0,u) = 2[K(x) - K(2L-x) - K(2L+x)] - (h_0/k)H_0(x) + (h_0/k)H_0(2L-x) + (h_0/k)H_0(2L+x)$ |
| 2A.X32 | $G_{X32}(x,0,u) = 2[K(x) + K(2L-x) + K(2L+x)] - (h_0/k)H_0(x) - (h_0/k)H_0(2L-x) - (h_0/k)H_0(2L+x)$ |
| 2A.X33a | $G_{X33}(x,0,u) = 2[K(x) + K(2L-x) + K(2L+x)] - (h_0/k)H_0(x) - (h_0/k)H_0(2L-x) - 2(h_L/k)H_L(2L-x) - (h_0/k)H_0(2L+x) - (h_L/k)H_L(2L+x) + JH(L,x,0)$ |
| | where for $h_0 \dots h_L$, |
| 2A.X33b | $JH(L,x,0) = (2h_0 h_L / (k(h_L - h_0))) [H_0(2L-x) + H_0(2L+x) - H_L(2L-x) - H_L(2L+x)]$ |
| | and for $h_0 = h_L$, |
| 2A.X33b | $JH(L,x,0) = (2h_0^2 / k^2) \{ 2(u/B)^{1/2} (\exp[-(2L-x)^2 / (4u)] + \exp[-(2L+x)^2 / (4u)]) - [2L-x + 2(h_0/k)u]H_0(2L-x) - [2L+x + 2(h_0/k)u]H_0(2L+x) \}$ |

If the surface temperature is given ($x = 0$ surface is nonhomogeneous), that is for X11, X12 and X13, we need not calculate the surface temperature since it is known. Notice

$$I = \int_{u=0}^t \frac{x}{u} K(x, u) du = \operatorname{erfc} \left[\frac{x}{(4\alpha t)^{1/2}} \right]$$

as x goes to 0, I goes to unity, which clearly is not zero. Also note that as x goes to 0, contributions to the integral only occurs for very small values of u . This means that

$$B = \lim_{x \rightarrow 0} I = \int_{u=0}^t \frac{x}{u} K(x, u) C(u) du = C(0)$$

Some limiting conditions:

- As h_i goes to zero, $H_i(z)$ goes to $E(z)$. As h_i goes to infinity, $H_i(z)$ goes to 0.
- As h_i goes to zero, $(h_i/k) H_i(z)$ goes to 0. As h_i goes to infinity, $(h_i/k) H_i(z)$ goes to $2K(z)$.
- As h_i goes to infinity, $(h_i/k)[(h_i/k) H_i(z) - 2K(z)]$ goes to $-2zK(z)/(2"u)$.

Table 2B. Derivative wrt x of Green's functions for boundary conditions of the 2nd and 3rd kinds and negative of mixed second partial derivative wrt x and n' of the Green's function for the 1st kind of boundary condition. Small time forms. All evaluated at $n=0$.

| | |
|---------|--|
| 2B.X10 | $-\dot{M}G_{X10}(x,0,u)/M = (u)^{-1}K(x)[1-x^2(2u)^{-1}]$ |
| 2B.X11 | $-\dot{M}G_{X11}(x,0,u)/M = (u)^{-1}[K(x)(1-x^2(2u)^{-1}) + K(2L-x)(1-(2L-x)^2(2u)^{-1}) + K(2L+x)(1-(2L+x)^2(2u)^{-1})]$ |
| 2B.X12 | $-\dot{M}G_{X12}(x,0,u)/M = (u)^{-1}[K(x)(1-x^2(2u)^{-1}) - K(2L-x)(1-(2L-x)^2(2u)^{-1}) - K(2L+x)(1-(2L+x)^2(2u)^{-1})]$ |
| 2B.X13 | $-\dot{M}G_{X13}(x,0,u)/M = (u)^{-1}[K(x)(1-x^2(2u)^{-1}) - K(2L-x)(1-(2L-x)^2(2u)^{-1}) - K(2L+x)(1-(2L+x)^2(2u)^{-1})] + 4(h_L/k)K(2L-x)[(h_L/k) - (2L-x)(2u)^{-1}] + 2(h_L/k)K(2L+x)[(h_L/k) - (2L+x)(2u)^{-1}] - 2(h_L/k)^3H_L(2L-x) - (h_L/k)^3H_L(2L+x)$ |
| 2B.X20 | $MG_{X20}(x,0,u)/M = -(u)^{-1}xK(x)$; see integral at bottom of Table 3.2A. |
| 2B.X21 | $MG_{X21}(x,0,u)/M = -(u)^{-1}[xK(x) + (2L-x)K(2L-x) - (2L+x)K(2L+x)]$ |
| 2B.X22 | $MG_{X22}(x,0,u)/M = -(u)^{-1}[xK(x) - (2L-x)K(2L-x) + (2L+x)K(2L+x)]$ |
| 2B.X23 | $MG_{X23}(x,0,u)/M = -(u)^{-1}[xK(x) - (2L-x)K(2L-x) + (2L+x)K(2L+x)] - 4(h_L/k)K(2L-x) + 2(h_L/k)^2H_L(2L-x) + 2(h_L/k)K(2L+x) - (h_L/k)^2H_L(2L+x)$ |
| 2B.X30 | $MG_{X30}(x,0,u)/M = -(u)^{-1}xK(x) + 2(h_0/k)K(x) - (h_0/k)^2H_0(x)$ |
| 2B.X31 | $MG_{X31}(x,0,u)/M = -(u)^{-1}[xK(x) + (2L-x)K(2L-x) - (2L+x)K(2L+x)] + 2(h_0/k)[K(x) + K(2L-x) - K(2L+x)] - (h_0/k)^2[H_0(x) + H_0(2L-x) - H_0(2L+x)]$ |
| 2B.X32 | $MG_{X32}(x,0,u)/M = -(u)^{-1}[xK(x) - (2L-x)K(2L-x) + (2L+x)K(2L+x)] + 2(h_0/k)[K(x) - K(2L-x) + K(2L+x)] - (h_0/k)^2[H_0(x) - H_0(2L-x) + H_0(2L+x)]$ |
| 2B.X33a | $MG_{X33}(x,0,u)/M = -(u)^{-1}[xK(x) - (2L-x)K(2L-x) + (2L+x)K(2L+x)] + 2(h_0/k)[K(x) - K(2L-x) + K(2L+x)] - (h_0/k)^2[H_0(x) - H_0(2L-x) + H_0(2L+x)] - 4(h_L/k)K(2L-x) + 2(h_L/k)K(2L+x) + (h_L/k)^2[2H_L(2L-x) - H_L(2L+x)] + DJH(L,x,0)$ |
| | where for $h_0 \dots h_L$, |
| 2B.X33b | $DJH(L,x,0) = (2h_0h_L/(k(h_L-h_0)))[-(h_0/k)(H_0(2L-x)-H_0(2L+x)) + (h_L/k)(H_L(2L-x)-H_L(2L+x))]$ |
| | and for $h_0 = h_L$, |
| 2B.X33c | $JH(L,x,0) = (2h_0^2/k^2)\{4(h_0/k)u[-K(2L-x) + K(2L+x)] + [1+(h_0/k)(2L-x) + 2(h_0/k)^2u]H_0(2L-x) - [1+(h_0/k)(2L+x) + 2(h_0/k)^2u]H_0(2L+x)\}$ |

Table 3A. Boundary Green's functions for boundary conditions of the 2nd and 3rd kinds and negative of the derivative of the Green's function with respect to n for the 1st kind of boundary condition. Small time forms and evaluated at $n = x = L$.

| Number | Equation |
|---------|---|
| 3A.X11 | $-\mathcal{M}_{X11}(x,L,u)/\mathcal{M}N. \quad (u)^{-1} [(L-x)K(L-x) - (L+x)K(L+x)], x \dots L$ |
| 3A.X12 | $G_{X12}(x,L,u) . \quad 2[K(L-x) - K(L+x)]$ |
| 3A.X13 | $G_{X13}(x,L,u) . \quad 2[K(L-x) - K(L+x)] - (h_L/k)H_L(L-x) + (h_L/k)H_L(L+x)$ |
| 3A.X21 | $-\mathcal{M}_{X21}(x,L,u)/\mathcal{M}N. \quad (u)^{-1} [(L-x)K(L-x) + (L+x)K(L+x)], x \dots L$ |
| 3A.X22 | $G_{X22}(x,L,u) . \quad 2[K(L-x) + K(L+x)]$ |
| 3A.X23 | $G_{X23}(x,L,u) . \quad 2[K(L-x) + K(L+x)] - (h_L/k)[H_L(L-x) + H_L(L+x)]$ |
| 3A.X31 | $-\mathcal{M}_{X31}(x,L,u)/\mathcal{M}N. \quad (u)^{-1} [(L-x)K(L-x) + (L+x)K(L+x)] - 4(h_0/k)K_0(L+x) + 2(h_0/k)^2 H_0(L+x), x \dots L$ |
| 3A.X32 | $G_{X32}(x,L,u) . \quad 2[K(L-x) + K(L+x) + K(3L-x)] - 2(h_0/k)H_0(L+x)$ |
| 3A.X33a | $G_{X33}(x,L,u) . \quad 2[K(L-x) + K(L+x) + K(3L-x)] - 2(h_0/k)H_0(L+x) - (h_L/k)H_L(L-x) - (h_L/k)H_L(L+x) + JH(L,x,L)$ where for $h_0 \dots h_L$ |
| 3A.X33b | $JH(L,x,L) . \quad (2h_0h_L/(k(h_L-h_0)))[H_0(L+x) - H_L(L+x)]$ and for $h_0 = h_L$ |
| 3A.X33c | $JH(L,x,x') . \quad (2h_0^2/k^2)[4(u)K(L+x) - (L+x + 2(h_0/k)u)H_0(L+x)]$ |

Table 3B. Derivative wrt x of Green's functions for boundary conditions of the 2nd and 3rd kinds and negative of Mixed Second Partial Derivative wrt x and n' of the Green's function for the 1st Kind of Boundary Condition. Small Time Forms and Evaluated at $nN=L$.

| Number | Equation |
|---------|--|
| 3B.X11 | $-\dot{M}G_{X11}(x,L,u)/\dot{M}M\dot{N}. -("u)^{-1}[K(L-x)(1-(L-x)^2(2 "u)^{-1}) + K(L+x)(1-(L+x)^2(2 "u)^{-1})]$ |
| 3B.X12 | $M\dot{G}_{X12}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) + (L+x)K(L+x)]$ |
| 3B.X13 | $M\dot{G}_{X13}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) + (L+x)K(L+x)] - 2(h_0/k)[K(L-x) + K(L+x)] + (h_0/k)^2[H_0(L-x) + H_0(L+x)]$ |
| 3B.X21 | $-\dot{M}G_{X21}(x,L,u)/\dot{M}M\dot{N}. -("u)^{-1}[K(L-x)(1-(L-x)^2(2 "u)^{-1}) - K(L+x)(1-(L+x)^2(2 "u)^{-1})]$ |
| 3B.X22 | $M\dot{G}_{X22}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) - (L+x)K(L+x)]$ |
| 3B.X23 | $M\dot{G}_{X23}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) - (L+x)K(L+x)] - 2(h_L/k)[K(L-x) - K(L+x) + (h_L/k)^2[H_L(L-x) - H_L(L+x)]$ |
| 3B.X31 | $-\dot{M}G_{X31}(x,L,u)/\dot{M}M\dot{N}. -("u)^{-1}[K(L-x)(1-(L-x)^2(2 "u)^{-1}) - K(L+x)(1-(L+x)^2(2 "u)^{-1})] - 4(h_0/k)K(L+x)[(h_L/k) - (L+x)(2 "u)^{-1}] + 2(h_L/k)^3H_L(L+x)$ |
| 3B.X32 | $M\dot{G}_{X32}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) - (L+x)K(L+x)] + 4(h_0/k)K(L+x) - 2(h_0/k)^2H_0(L+x)$ |
| 3B.X33a | $M\dot{G}_{X33}(x,L,u)/\dot{M}. (" u)^{-1}[(L-x)K(L-x) - (L+x)K(L+x)] + 4(h_0/k)K(L+x) - 2(h_0/k)^2H_0(L+x) - 2(h_L/k)[K(L-x) - K(L+x)] + (h_L/k)^2[H_L(L-x) - H_L(L+x)] + DJH(L,x,L)$ |

where for $h_0 \dots h_L$,

$$3B.X33a \quad DJH(L,x,L) = (2h_0h_L/(k(h_L-h_0)))[(h_0/k)(H_0(L+x) - (h_L/k)(H_L(L+x))]$$

and for $h_0 = h_L$,

$$3B.X33b \quad DJH(L,x,L) = (2h_0^2/k^2)\{4(h_0/k) "uK(L+x) - [1+(h_0/k)(L+x) + 2(h_0/k)^2 "u]H_0(L+x)\}$$

Table 4A. Integrals of small time form of Green's functions, from $x \in [0, L]$ for the finite cases and from $x \in [0, 4]$ for the X10 cases. The X10, X20, X22 and X30 results are exact. Note that $\text{erfc}(2/(4 \cdot 0.05)^{1/2}) \approx 2.54E-10$ and $\text{erfc}(3/(4 \cdot 0.05)^{1/2}) \approx 2.38E-21$ so for dimensionless times less than 0.05 the $E(2L+x)$ and $E(3L-x)$ terms can be dropped. Note that we define

Also we observe that $H_i(z) \neq E(z)$ for $z > 0$. As $u \rightarrow 0$ and x is nonzero, $E(x) \rightarrow 0$ and $H(x) \rightarrow 0$. For every boundary condition except the first, the integrated Green's function goes to unity as $u \rightarrow 0$ for all x .

| Number | Equation |
|---------|---|
| 4A.X10 | $\int_0^L G_{X10}(x, x) \sqrt{u} dx = 1 - E(x) = \text{erf}[x/(4u)^{1/2}]$ |
| 4A.X11 | $\int_0^L G_{X11}(x, x) \sqrt{u} dx = 1 - E(x) - E(L-x) + E(L+x) + E(2L-x)$ |
| 4A.X12 | $\int_0^L G_{X12}(x, x) \sqrt{u} dx = 1 - E(x) - E(2L-x)$ |
| 4A.X13 | $\int_0^L G_{X13}(x, x) \sqrt{u} dx = 1 - E(x) - E(L-x) + E(L+x) + E(2L-x) + H_L(L-x) - H_L(L+x) - 2H_L(2L-x)$ |
| 4A.X20 | $\int_0^L G_{X20}(x, x) \sqrt{u} dx = 1$ |
| 4A.X21 | $\int_0^L G_{X21}(x, x) \sqrt{u} dx = 1 - E(L-x) - E(L+x)$ |
| 4A.X22 | $\int_0^L G_{X22}(x, x) \sqrt{u} dx = 1$ |
| 4A.X23 | $\int_0^L G_{X23}(x, x) \sqrt{u} dx = 1 - E(L-x) - E(L+x) + H_L(L-x) + H_L(L+x)$ |
| 4A.X30 | $\int_0^L G_{X30}(x, x) \sqrt{u} dx = 1 - E(x) + H_0(x)$ |
| 4A.X31 | $\int_0^L G_{X31}(x, x) \sqrt{u} dx = 1 - E(x) - E(L-x) + E(L+x) + E(2L-x) + H_0(x) - 2H_0(L+x) - H_0(2L-x)$ |
| 4A.X32 | $\int_0^L G_{X32}(x, x) \sqrt{u} dx = 1 - E(x) - E(2L-x) + H_0(x) + H_0(2L-x)$ |
| 4A.X33a | $\int_0^L G_{X33}(x, x) \sqrt{u} dx = 1 - E(x) - E(L-x) - E(L+x) - E(2L-x) + H_0(x) + H_0(2L-x) + H_L(L+x) + H_L(L-x) + IJH(L, x)$ |
| | where for $h_0 \dots h_L$ |
| 4A.X33b | $IJH(L, x) = 2E(L+x) + 2E(2L-x) - (2h_L/(h_L-h_0))[H_0(2L-x) + H_0(L+x)] + (2h_0/(h_L-h_0))[H_L(2L-x) + H_L(L+x)]$ |
| | and where for $h_0 = h_L = h$ |
| 4A.X33c | $IJH(L, x) = 2E(L+x) + 2E(2L-x) + 2H_0(2L-x)[-1 + 2\sqrt{u}(h/k)^2 + h(2L-x)/k] + 2H_0(L+x)[-1 + 2\sqrt{u}(h/k)^2 + h(L+x)/k] - (8h\sqrt{u}/k)[K(2L-x) + K(L+x)]$ |

Table 4B. Derivative wrt x of integrals of small time form of Green's functions, from $x=0$ to L .

| Number | Equation |
|---------|--|
| 4B.X11 | $d[\int_0^L G_{X11}(x,x)u dx]/dx = 2[K(x) - K(L-x) - K(L+x) + K(2L-x)]$ |
| 4B.X12 | $d[\int_0^L G_{X12}(x,x)u dx]/dx = 2[K(x) - K(2L-x)]$ |
| 4B.X13 | $d[\int_0^L G_{X13}(x,x)u dx]/dx = 2[K(x) - K(2L-x)] - h_L/k[H_L(L-x) + H_L(L+x) - 2 H_L(2L-x)]$ |
| 4B.X21 | $d[\int_0^L G_{X21}(x,x)u dx]/dx = -2K(L-x) + 2K(L+x)$ |
| 4B.X22 | $d[\int_0^L G_{X22}(x,x)u dx]/dx = 0$ |
| 4B.X23 | $d[\int_0^L G_{X23}(x,x)u dx]/dx = -h_L/k[H_L(L-x) - H_L(L+x)]$ |
| 4B.X31 | $d[\int_0^L G_{X31}(x,x)u dx]/dx = -2[K(L-x) - K(L+x)] + h_0/k[H_0(x) - 2H_0(L+x) + H_0(2L-x)]$ |
| 4B.X32 | $d[\int_0^L G_{X32}(x,x)u dx]/dx = h_0/k[H_0(x) - H_0(2L-x)]$ |
| 4B.X33a | $d[\int_0^L G_{X33}(x,x)u dx]/dx = h_0/k[H_0(x) - H_0(2L-x)] - h_L/k[H_L(L-x) - H_L(L+x)] + DIJH$ where for $h_0 \dots h_L$ |
| 4B.X33b | $DIJH = 2h_0h_L/k(h_L-h_0)[-H_0(L+x) + H_0(2L-x) + H_L(L+x) - H_L(2L-x)]$ and for $h_0 = h_L = h$ |
| 4B.X33c | $DIJH = 2h^2/k^2 \{ 4 \int_0^L u [-K(L+x) + K(2L-x)] - (L+x + 2h \int_0^L u/k)H(L+x) - (2L-x + 2h \int_0^L u/k)H(2L-x) \}$ |

