Standard Process Inputs
1st Order Systems

	Laplace Domain	Time Domain
ODE	$\dfrac{\bar{y}}{\bar{f}} = \dfrac{K}{\tau s + 1}$	$\tau \dfrac{dy}{dt} + y = K \cdot f(t)$
Impulse Function $f(t) = M \cdot \delta(t) \;\Rightarrow\; \bar{f}(s) = M$	$\bar{y} = \dfrac{MK}{\tau s + 1}$	$y = \dfrac{MK}{\tau} e^{-t/\tau}$
Step Function $f(t) = M \cdot S(t) \;\Rightarrow\; \bar{f}(s) = \dfrac{M}{s}$	$\bar{y} = \dfrac{MK}{s(\tau s + 1)}$	$y = MK\left(1 - e^{-t/\tau}\right)$
Ramp Function $f(t) = mt \;\Rightarrow\; \bar{f}(s) = \dfrac{m}{s^2}$	$\bar{y} = \dfrac{mK}{s^2(\tau s + 1)}$	$y = mK\left(t - \tau + \tau e^{-t/\tau}\right)$
Sinusoidal Function $f(t) = A\sin(\omega t) \;\Rightarrow\; \bar{f}(s) = \dfrac{A\omega}{s^2 + \omega^2}$	$\bar{y} = \dfrac{A\omega K}{(\tau s + 1)(s^2 + \omega^2)}$	$y = \dfrac{AK}{1 + \tau^2\omega^2}\left(\omega\tau e^{-t/\tau} - \omega\tau\cos\omega t + \sin\omega t\right)$ $= \dfrac{AK\omega\tau}{1 + \tau^2\omega^2} e^{-t/\tau} + \dfrac{AK}{\sqrt{1 + \tau^2\omega^2}} \sin[\omega t + \phi], \quad \tan\phi \equiv -\omega\tau$

Standard Process Inputs
2^{nd} Order Systems

	Laplace Domain	Time Domain
ODE	$\frac{\bar{y}}{\bar{f}} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\tau^2 \frac{d^2 y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y = K \cdot f(t)$
Impulse Function $f(t) = M \cdot \delta(t)$ $\Rightarrow \quad \bar{f}(s) = M$	$\bar{y} = \frac{MK}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\zeta > 1: \quad y = MK \frac{1}{\tau\sqrt{\zeta^2 - 1}} e^{-t\zeta/\tau} \sinh\left(\frac{\sqrt{\zeta^2-1}}{\tau} t\right)$ $= MK \frac{1}{\tau\sqrt{\zeta^2-1}} \cdot \frac{1}{2} \cdot \left[\exp\left(-\frac{\zeta - \sqrt{\zeta^2-1}}{\tau} t\right) - \exp\left(-\frac{\zeta + \sqrt{\zeta^2-1}}{\tau} t\right)\right]$ $\zeta = 1: \quad y = MK \frac{t}{\tau^2} e^{-t/\tau}$ $\zeta < 1: \quad y = MK \frac{1}{\tau\sqrt{1-\zeta^2}} e^{-t\zeta/\tau} \sin(\omega t), \quad \omega \equiv \frac{\sqrt{1-\zeta^2}}{\tau}$
Step Function $f(t) = M \cdot S(t)$ $\Rightarrow \quad \bar{f}(s) = \frac{M}{s}$	$\bar{y} = \frac{MK}{s(\tau^2 s^2 + 2\zeta\tau s + 1)}$	$\zeta > 1: \quad y = MK \left\{1 - e^{-t\zeta/\tau}\left[\cosh\left(\frac{\sqrt{\zeta^2-1}}{\tau} t\right) + \frac{\zeta}{\sqrt{\zeta^2-1}} \sinh\left(\frac{\sqrt{\zeta^2-1}}{\tau} t\right)\right]\right\}$ $= MK \left\{1 - \frac{1}{2}\left[\left(1 + \frac{\zeta}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta - \sqrt{\zeta^2-1}}{\tau} t\right) + \left(1 - \frac{\zeta}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta + \sqrt{\zeta^2-1}}{\tau} t\right)\right]\right\}$ $\zeta = 1: \quad y = MK\left[1 - \left(1 + \frac{t}{\tau}\right)e^{-t/\tau}\right]$ $\zeta < 1: \quad y = MK\left\{1 - e^{-t\zeta/\tau}\left[\cos(\omega t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega t)\right]\right\}$ $= MK\left[1 - \frac{e^{-t\zeta/\tau}}{\sqrt{1-\zeta^2}}\sin(\omega t + \phi)\right], \quad \omega \equiv \frac{\sqrt{1-\zeta^2}}{\tau}, \quad \tan\phi \equiv \frac{\sqrt{1-\zeta^2}}{\zeta}$

Standard Process Inputs
2nd Order Systems

	Laplace Domain	Time Domain
Ramp Function $f(t)=mt$ $\Rightarrow \quad \bar{f}(s)=\frac{m}{s^2}$	$\bar{y}=\frac{mK}{s^2\left(\tau^2 s^2+2\zeta\tau s+1\right)}$	For all cases: $y_\infty(t)=mK(t-2\zeta\tau)$ $\zeta>1: \quad y=y_\infty+mK\tau e^{-t\zeta/\tau}\left[2\zeta\cosh\left(\frac{\sqrt{\zeta^2-1}}{\tau}t\right)+\frac{2\zeta^2-1}{\sqrt{\zeta^2-1}}\sinh\left(\frac{\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $=y_\infty+\frac{mK\tau}{2}\left[\left(2\zeta+\frac{2\zeta^2-1}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta-\sqrt{\zeta^2-1}}{\tau}t\right)+\left(2\zeta-\frac{2\zeta^2-1}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta+\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $\zeta=1: \quad y=y_\infty+mK(2\tau+t)e^{-t/\tau}$ $\zeta<1: \quad y=y_\infty+mK\tau e^{-t\zeta/\tau}\left[2\zeta\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)+\frac{2\zeta^2-1}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]$ $=y_\infty+\frac{mK\tau e^{-t\zeta/\tau}}{\sqrt{1-\zeta^2}}\sin(\omega t+\phi), \quad \omega\equiv\frac{\sqrt{1-\zeta^2}}{\tau}, \quad \tan\phi\equiv\frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}$
	$\bar{y}=\frac{s}{\tau^2 s^2+2\zeta\tau s+1}$	$\zeta>1: \quad y=\frac{e^{-t\zeta/\tau}}{\tau^2}\left[\cosh\left(\frac{\sqrt{\zeta^2-1}}{\tau}t\right)-\frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\left(\frac{\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $=\frac{1}{2\tau^2}\left[\left(1-\frac{\zeta}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta-\sqrt{\zeta^2-1}}{\tau}t\right)+\left(1+\frac{\zeta}{\sqrt{\zeta^2-1}}\right)\exp\left(-\frac{\zeta+\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $\zeta=1: \quad y=\frac{e^{-t/\tau}}{\tau^2}\left(1-\frac{t}{\tau}\right)$ $\zeta<1: \quad y=\frac{e^{-t\zeta/\tau}}{\tau^2}\left[\cos\left(t\frac{\sqrt{1-\zeta^2}}{\tau}\right)-\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(t\frac{\sqrt{1-\zeta^2}}{\tau}\right)\right]$ $=\frac{e^{-t\zeta/\tau}}{\tau^2\sqrt{1-\zeta^2}}\sin(\omega t+\phi), \quad \omega\equiv\frac{\sqrt{1-\zeta^2}}{\tau} \quad \tan\phi\equiv-\frac{\sqrt{1-\zeta^2}}{\zeta}$

Standard Process Inputs
2nd Order Systems

	Laplace Domain	Time Domain
Sinusoidal Function $f(t) = A\sin(\omega t)$ $\Rightarrow \quad \bar{f}(s) = \dfrac{A\omega}{s^2+\omega^2}$	$\bar{y} = \dfrac{A\omega K}{\left(\tau^2 s^2 + 2\zeta\tau s + 1\right)\left(s^2+\omega^2\right)}$	For all cases: $\mathsf{F} \equiv \omega^4\tau^4 - 2\omega^2\tau^2 + 1 + 4\omega^2\tau^2\zeta^2 = \left(1-\omega^2\tau^2\right)^2 + \left(2\omega\zeta\tau\right)^2$ $y_\infty(t) = \dfrac{AK}{\mathsf{F}}\left[\left(1-\omega^2\tau^2\right)\sin(\omega t) - 2\omega\zeta\tau\cos(\omega t)\right]$ $= \dfrac{AK}{\sqrt{\left(2\omega\zeta\tau\right)^2 + \left(1-\omega^2\tau^2\right)^2}}\sin\left(\omega t + \phi_\infty\right), \quad \tan\phi_\infty \equiv -\dfrac{2\omega\zeta\tau}{1-\omega^2\tau^2}$ $\zeta > 1: \quad y = y_\infty + AK\dfrac{\omega\tau}{\mathsf{F}}e^{-t\zeta/\tau}\left[2\zeta\cosh\left(\dfrac{\sqrt{\zeta^2-1}}{\tau}t\right) + \dfrac{\omega^2\tau^2+2\zeta^2-1}{\sqrt{\zeta^2-1}}\sinh\left(\dfrac{\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $= y_\infty + AK\dfrac{\omega\tau}{2\mathsf{F}}\left[\left(2\zeta + \dfrac{\omega^2\tau^2+2\zeta^2-1}{\sqrt{\zeta^2-1}}\right)\exp\left(-\dfrac{\zeta-\sqrt{\zeta^2-1}}{\tau}t\right) + \left(2\zeta - \dfrac{\omega^2\tau^2+2\zeta^2-1}{\sqrt{\zeta^2-1}}\right)\exp\left(-\dfrac{\zeta+\sqrt{\zeta^2-1}}{\tau}t\right)\right]$ $\zeta = 1: \quad y = y_\infty + AK\dfrac{\omega}{\mathsf{F}}e^{-t\zeta/\tau}\left[2\tau + \left(\omega^2\tau^2+1\right)t\right]$ $\zeta < 1: \quad y = y_\infty + AK\dfrac{\omega\tau}{\mathsf{F}}e^{-t\zeta/\tau}\left[2\zeta\cos\left(\dfrac{\sqrt{1-\zeta^2}}{\tau}t\right) + \dfrac{\omega^2\tau^2+2\zeta^2-1}{\sqrt{1-\zeta^2}}\sin\left(\dfrac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]$ $= y_\infty + AK\dfrac{\omega\tau}{\sqrt{1-\zeta^2}\sqrt{\left(\omega^2\tau^2-1\right)^2 + \left(2\omega\zeta\tau\right)^2}}e^{-t\zeta/\tau}\sin\left(\omega_2 t + \phi_2\right)$ $\omega_2 \equiv \dfrac{\sqrt{1-\zeta^2}}{\tau}, \quad \tan\phi_2 \equiv \dfrac{2\zeta\sqrt{1-\zeta^2}}{\omega^2\tau^2+2\zeta^2-1}$