

Design & Tuning of Feedback Control Systems

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The preceding topics raise the questions:

- How do we select an appropriate type of feedback control scheme?
- How do we adjust the parameters to achieve an “optimum” response?

Appropriate Feedback Control Scheme

The following rules of thumb are given in the SEM (2nd edition) text. The recommendations are based upon the type of variable to be controlled.

- **Flow rate, use PI control** with typical controller settings of $0.5 < K_c < 0.7$ and $0.2 < \tau_i < 0.3$ min .
- **Liquid level, use PI control** with typical controller settings of

$$K_c = \frac{1}{\Delta h} \quad \text{where } \Delta h = \min(h_{\max} - h_{sp}, h_{sp} - h_{\min})$$

$$\tau_i = \frac{4V}{K_c F_{\max}} \quad \text{where } \Delta h = \min(h_{\max} - h_{sp}, h_{sp} - h_{\min})$$

- **Gas pressure use PI control.**
- **Temperature, use PI or PID control.**
- **Composition use PI or advanced control.**

Controller Tuning Criteria

There are several “qualitative” tuning criteria:

- Return to desired level operation as soon as possible.
- Keep the maximum deviation as small as possible.

Can distinguish between steady state & dynamic performance criteria:

- Zero offset at steady state.
- Use only a few points of the response (simple dynamic).
- Use all of the points of the response (complex dynamic).

Dynamic Controller Tuning Criteria

Based on simple statistics: overshoot, rise time, decay ratio, etc. Most popular have tended to be developed to give a $\frac{1}{4}$ **decay ratio**.

More complex tuning criteria are based upon minimizing the total error of the response. Three types can be defined:

- Integral of the square error (ISE):

$$ISE = \int_0^{\infty} \varepsilon^2(t) dt$$

Strongly suppresses large errors.

- Integral of the absolute value of the error (IAE):

$$IAE = \int_0^{\infty} |\varepsilon(t)| dt$$

Strongly suppresses small errors.

- Integral of the time-weighted absolute error (ITAE):

$$ITAE = \int_0^{\infty} t |\varepsilon(t)| dt$$

Strongly suppresses errors that persist for long times.

In actual practice most professionals aim for critically damped tuning. All of the above reward overshoot, whereas in the real world overshoot ends up as oscillation in downstream processes or in interacting loops and also reduces your stability margin. Since real world loops are non-linear, you get into much less trouble if you build in excess stability.

Controller Tuning Methods

Initial controller settings can be made using a variety of approaches. The following will be discussed in this class:

- “Rule of Thumb” settings.
- Settings based upon response of the system to an open-loop or closed-loop disturbance.
- Settings based upon the desired response of the system to a set point or load disturbance.

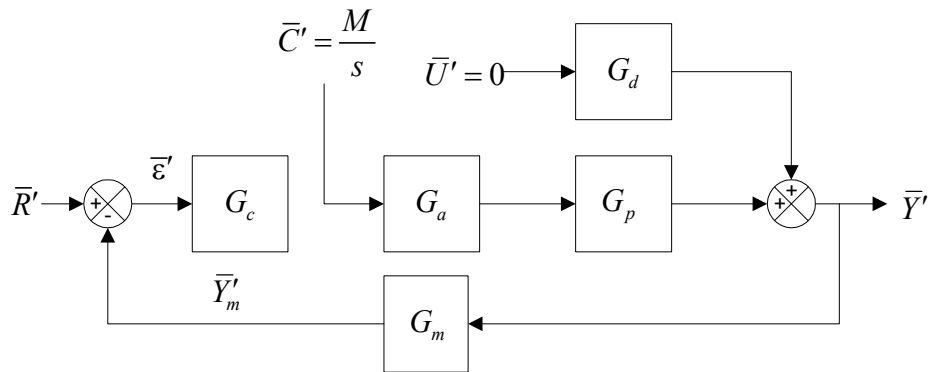
“Rule of Thumb” Tuning

A typical approach for tuning PID controllers is as follows:

- Eliminate integral & derivative action in the controller – i.e., go on P control.
- Set K_c at a low value (e.g., 0.5) & put the controller on automatic.
- Increase the controller gain by small increments continuous cycling occurs after a small set point or load change. Reduce K_c by a factor of 2.
- Decrease τ_I by small increments until continuous cycling occurs again. Set τ_I to 3 times this value.
- Increase τ_D until continuous cycling occurs. Set τ_D to 1/3 this value.

“Continuous cycling” is defined as having sustained oscillations with constant amplitude. The value of the controller gain at this point is the ultimate gain, K_{cu} . The period of oscillation will be T_u or P_u .

Controller Tuning With Process Reaction Curve Method



The Cohen & Coon method applies only to open-loop processes that are inherently stable. Cohen & Coon suggest method to first model process in the open loop & then picking the appropriate control parameters. Noted that most responses have a “sigmoidal” response to step change. They suggest first modeling as 1st order process with dead time:

$$G_{PRC}(s) = \frac{\bar{Y}'_m}{\bar{C}'} = G_a G_p G_m \approx \frac{K e^{-\theta s}}{\tau s + 1}$$

This is what they called the **process reaction** curve (PRC). Then they used various performance criteria:

- 1/4 decay ratio.
- Minimum offset.
- Minimum integral of the square error (ISE) under the load response curve.

They came up with the criteria in the following table.

Type of Control	K_c	τ_I	τ_D
P	$\frac{1}{K} \frac{\tau}{\theta} \left(1 + \frac{\theta}{3\tau} \right)$	—	—
PI	$\frac{1}{K} \frac{\tau}{\theta} \left(0.9 + \frac{\theta}{12\tau} \right)$	$\theta \frac{30 + 3(\theta/\tau)}{9 + 20(\theta/\tau)}$	—
PID	$\frac{1}{K} \frac{\tau}{\theta} \left(\frac{4}{3} + \frac{\theta}{4\tau} \right)$	$\theta \frac{32 + 6(\theta/\tau)}{13 + 8(\theta/\tau)}$	$\theta \frac{4}{11 + 2(\theta/\tau)}$

Ziegler-Nichols Controller Tuning

The Cohen & Coon method uses open loop responses as a basis for controller tuning. The Ziegler-Nichols method is based upon closed-loop response instead of an open-loop response. However, it is based upon frequency response methods (which we will not be able to cover this semester). The ZN method is more widely used in the industry.

Ed prefers Z-N tuning to Cohen & Coon. It is not often possible to do an open loop step test on a loop. Note that the Z-N settings are very aggressive. Probably the best thing you can do for your class is set them down with a simulation and let them tune loops.

The steps are as follows:

- Bring system to steady state operation.
- Put on P control. Introduce a set point change and vary gain until system oscillates continuously. This frequency is ω_{CO} and M is the amplitude ratio.
- Compute the following:

$$\text{Ultimate Gain} = K_u = \frac{1}{M}$$

$$\text{Ultimate Period} = P_u = \frac{2\pi}{\omega_{CO}}$$

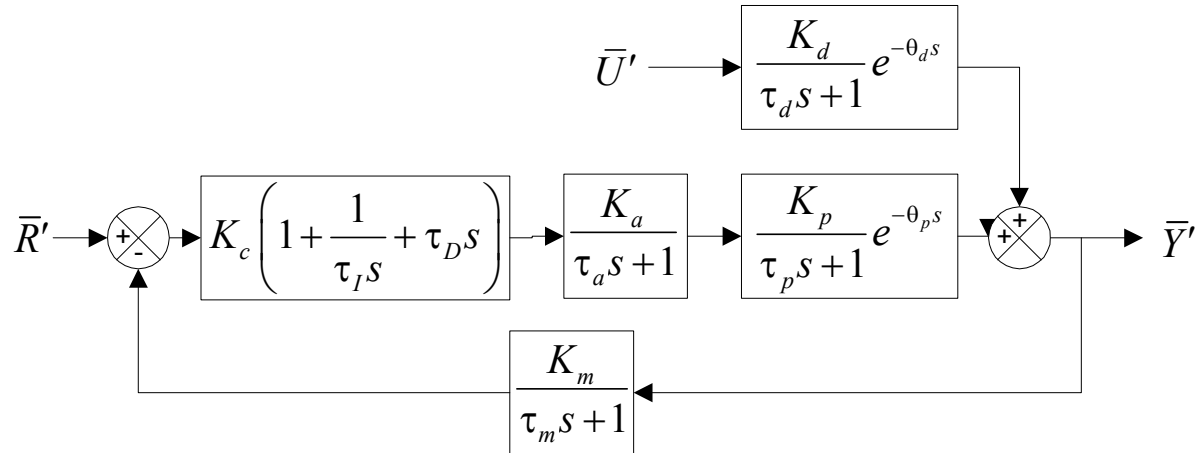
The original Z-N tuning settings are given in the following table.

Type of Control	K_c	τ_I	τ_D
P	$K_u/2$	—	—
PI	$K_u/2.2$	$P_u/1.2$	—
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

These controller settings were developed to give a $1/4$ decay ratio. However, other settings have been recommended that are closer to critically damped control (so that oscillations do not propagate downstream). PI & PID controller settings suggested by Tyreus & Luyben are shown in the following table.

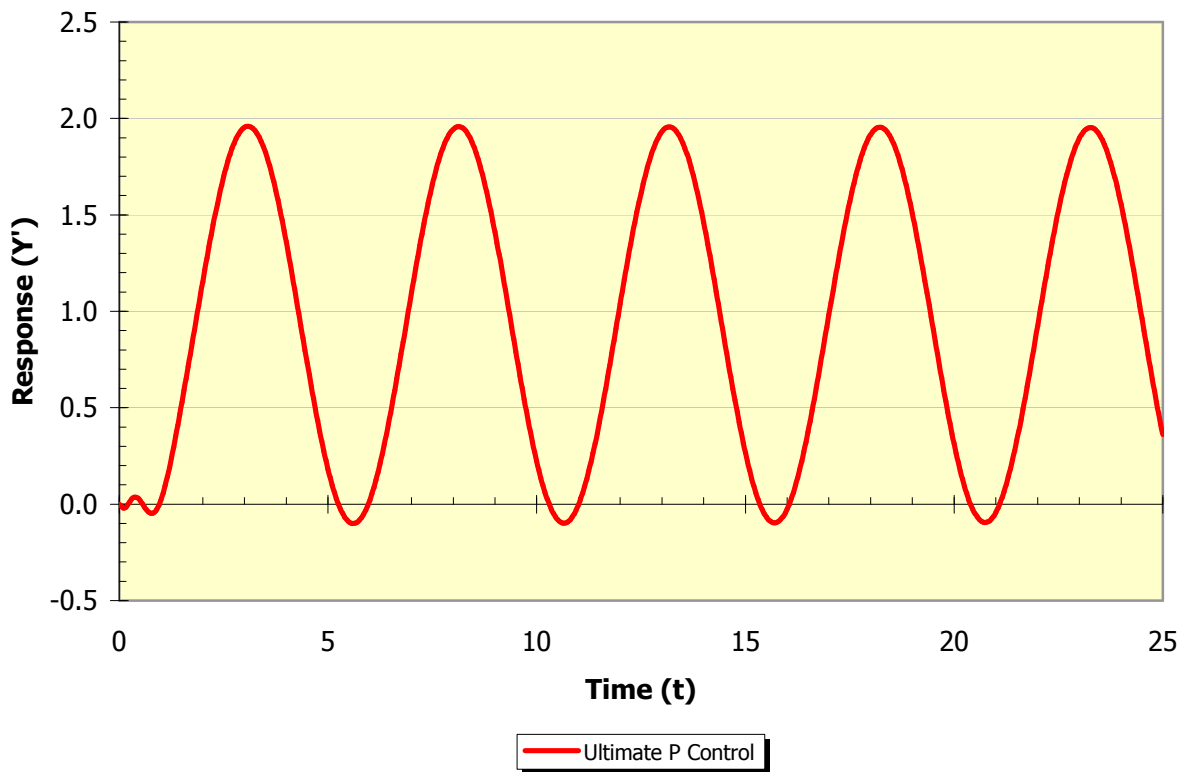
Type of Control	K_c	τ_I	τ_D
PI	$K_u/3.2$	$2.2P_u$	—
PID	$K_u/2.2$	$2.2P_u$	$P_u/6.3$

Example

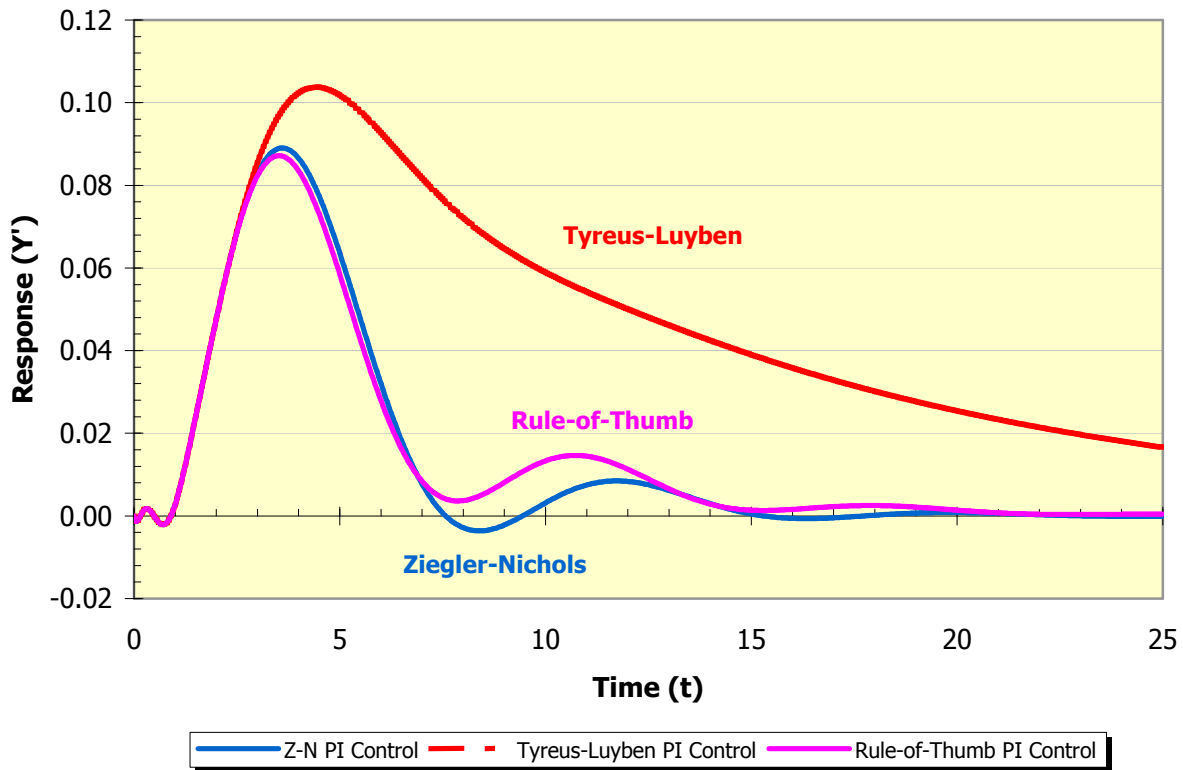


Let's look at the above process as an example of the response to Ziegler-Nichols tuning parameters. The calculations will be done using *Mathematica* and employ a 3rd order Padé approximation for the time delays. The following table will give the process parameters to be used.

Parameter	Value	Parameter	Value
$K_d = K_p$	0.5	K_a	1
$\tau_d = \tau_p$	10 min.	τ_a	5 sec.
$\theta_d = \theta_p$	1 min.	K_m	1
		τ_m	15 sec.



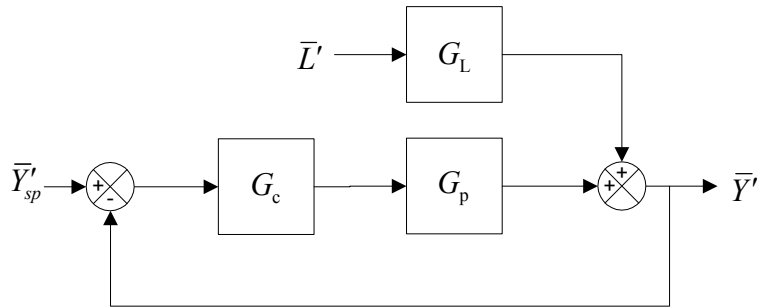
Using a trial-and-error procedure the ultimate gain (that leads to stable oscillations) is about 26.3 (see the following response curve). The period of oscillation is estimated to 5.0 (as calculated from peak to peak).



This figure shows three different response curves to unit step load disturbances. The first is for the original PI control ($K_c = 12.0$ & $\tau_I = 4.2$), the second is for the Tyreus-Luyben settings ($K_c = 8.2$ & $\tau_I = 11.4$), & the third is for the Rule-of-Thumb adjustment ($K_c = 13.15$ & $\tau_I = 5.1$). Notice that there is little difference between the response curves from the Zeigler-Nichols & Rule-of-Thumb settings. Further notice that though there is no oscillation with the Tyreus-Luyben settings it has a much greater maximum deviation & a much slower response time.

Direct Synthesis Controller Tuning

Direct synthesis methods are based upon prescribing a desired form for the system's response and then finding a controller strategy & parameters to give that response. Our focus can either be on rejecting set point disturbances or load disturbances.



For the feedback control loop above the overall transfer functions between the output \bar{Y}' and the set point & disturbance are:

$$\frac{\bar{Y}'}{\bar{Y}'_{sp}} = \frac{G_c G_p}{1 + G_c G_p} \quad \text{and} \quad \frac{\bar{Y}'}{\bar{L}'} = \frac{G_L}{1 + G_c G_p}.$$

Note that for the set point transfer function, we can manipulate it to give:

$$G_c = \frac{(\bar{Y}'/\bar{Y}'_{sp})}{G_p [1 - (\bar{Y}'/\bar{Y}'_{sp})]}.$$

One implication is that if we pick a desired form for the response to a set point change, \bar{Y}'/\bar{Y}'_{sp} , then we have set out the desired form for the controller. For example, we might think that it would be great to have the output to immediately track the set point change, i.e., $\bar{Y}' = \bar{Y}'_{sp}$. However, doing this would require an infinite gain in the controller:

$$G_c = \frac{1}{G_p} \cdot \frac{1}{0} \rightarrow \infty.$$

A more practical response would be a first order decay into the final value, or:

$$\frac{\bar{Y}'}{\bar{Y}'_{sp}} = \frac{1}{\tau_c s + 1}.$$

If we require this type of response for a step change disturbance then we get a controller strategy with the form:

$$G_c = \frac{\left(\frac{1}{\tau_c s + 1} \right)}{G_p \left[1 - \left(\frac{1}{\tau_c s + 1} \right) \right]} = \frac{1}{G_p} \cdot \frac{1}{\tau_c s}.$$

Notice that this shows there is an integral action to the Direct Synthesis controller strategy.

First Order Process

Let's look at the Direct Synthesis controller strategy for a first order process:

$$G_p = \frac{K_p}{\tau_p s + 1}.$$

Then the controller strategy is:

$$G_c = \frac{1}{\frac{K_p}{\tau_p s + 1}} \cdot \frac{1}{\tau_c s} = \frac{\tau_p s + 1}{K_p \tau_c s} = \frac{\tau_p}{K_p \tau_c} \left(1 + \frac{1}{\tau_p s} \right).$$

Notice that this is simply PI control with the settings:

$$K_c = \frac{\tau_p}{K_p \tau_c} \quad \text{and} \quad \tau_i = \tau_p.$$

Notice that the controller settling time τ_c is only in the controller gain K_c & the integral time τ_i is only based on the process parameter τ_p .

First Order Process with Dead Time

Applying the Direct Synthesis procedure to a process with dead time will require some type of approximation to the dead time term to be able to end up with a controller strategy in a PID form. Let's also look at the Direct Synthesis controller strategy applied to an FOPDT process. For this process the transfer function is:

$$G_p = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}.$$

Now it makes more sense to require that the response to a set point disturbance should also have a time delay that matches the process's time delay:

$$\frac{\bar{Y}'}{\bar{Y}'_{sp}} = \frac{e^{-\theta_p s}}{\tau_c s + 1}.$$

Now the controller strategy implied by Direct Synthesis is:

$$G_c = \frac{\left(\frac{e^{-\theta_p s}}{\tau_c s + 1} \right)}{G_p \left[1 - \left(\frac{e^{-\theta_p s}}{\tau_c s + 1} \right) \right]} = \frac{1}{G_p} \cdot \frac{e^{-\theta_p s}}{\tau_c s + 1 - e^{-\theta_p s}}$$

and for our FOPDT process:

$$G_c = \frac{\tau_p s + 1}{K_p e^{-\theta_p s}} \cdot \frac{e^{-\theta_p s}}{\tau_c s + 1 - e^{-\theta_p s}} = \frac{\tau_p s + 1}{K_p} \cdot \frac{1}{\tau_c s + 1 - e^{-\theta_p s}}.$$

The time delay term cannot be physically realized. If we do a simple Taylor series expansion for the time delay term, $e^{-\theta_p s} \approx 1 - \theta_p s$, then we get:

$$G_c = \frac{\tau_p s + 1}{K_p} \cdot \frac{1}{\tau_c s + 1 - (1 - \theta_p s)} = \frac{\tau_p s + 1}{K_p} \cdot \frac{1}{(\tau_c + \theta_p) s} = \frac{\tau_p}{K_p (\tau_c + \theta_p)} \cdot \left(1 + \frac{1}{\tau_p s} \right).$$

We still have PI control but now the parameters are:

$$K_c = \frac{\tau_p}{K_p (\tau_c + \theta_p)} \quad \text{and} \quad \tau_I = \tau_p.$$

We used a simple truncated Taylor series expansion for the dead time term here. What happens if we use a Padé approximation instead? We can show that when using a 1st order Pade approximation and assuming that the dead time is much less than both the controller settling time ($\theta_p \ll \tau_c$) and the process time constant ($\theta_p \ll \tau_p$) then the controller strategy for a FOPDT is:

$$G_c = \frac{\tau_p}{K_p (\tau_c + \theta_p)} \left[1 + \frac{1}{\tau_p s} + \frac{\theta_p}{2} s \right]$$

and this is now in the form of a PID controller with the settings:

$$K_c = \frac{\tau_p}{K_p(\tau_c + \theta_p)}, \tau_I = \tau_p, \text{ and } \tau_D = \frac{1}{2}\theta_p.$$

Summary of Controller Settings Using Direct Synthesis for Rejection of Set Point Disturbances

SEM (2nd edition) and Smith & Corropio have summarized the PID settings for various order processes. Many of these are presented in the following table. SM recommend that if $\theta_p > \tau_p / 4$ in an FOPDT then PID control should be used, not PI control.

$G_p(s)$	Type of Control	K_c	τ_I	τ_D
K_p	I	—	$\frac{1}{K_p \tau_c}$	—
$\frac{K_p}{\tau_p s + 1}$	PI	$\frac{\tau_p}{K_p \tau_c}$	τ_p	—
$\frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$	PID	$\frac{\tau_1 + \tau_2}{K_p \tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
$\frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}, \tau_1 \gg \tau_2$	PID	$\frac{\tau_1}{K_p \tau_c}$	τ_1	τ_2
$\frac{K_p}{\tau^2 s^2 + 2\zeta \tau s + 1}$	PID	$\frac{2\zeta \tau}{K_p \tau_c}$	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
$\frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$	PI	$\frac{\tau_p}{K_p(\tau_c + \theta_p)}$	τ_p	—
$\frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$ (SEM)	PID	$\frac{\tau_p + \frac{\theta_p}{2}}{K_p \left(\tau_c + \frac{\theta_p}{2} \right)}$	$\tau_p + \frac{\theta_p}{2}$	$\frac{\tau_p \theta_p}{2\tau_p + \theta_p}$
$\frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$ (SC)	PID	$\frac{\tau_p}{K_p(\tau_c + \theta_p)}$	τ_p	$\frac{1}{2}\theta_p$
$\frac{K_p}{s}$	P	$\frac{1}{K_p \tau_c}$	—	—