Use these matrices for problems 1 and 2:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, \; C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \; D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}, \; E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

1. If possible, compute
   (a) $A(BD)$
   (b) $(AB)D$
   (c) $A(C + E)$
   (d) $AC + AE$
   (e) $3A + 2A$
   (f) $5A$

2. If possible, compute
   (a) A^T
   (b) $(A^T)^T$
   (c) $(AB)^T$
   (d) $B^T A^T$
   (e) $(C + E)^T$
   (f) $C^T + E^T$

3. By hand (that is, without using your calculator), find the inverse of $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$. That is, find a 2×2 matrix B such that $AB = BA = I_2$.

4. Write the matrix $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 2 & 3 \\ -1 & 6 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

5. (challenge) Find two different 2×2 matrices A having no zero entries such that $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.