

Solution to Problem 10207
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Summing a Series of Volumes

10207. *Proposed by Eric Freden (student), Brigham Young University, Provo, UT.*

Find a closed form for $\sum_{n=0}^{\infty} \text{Vol}(B^n)$ where B^n is the unit ball in \mathbb{R}^n (and $\text{Vol}(B^0)$ is taken to be 1).

SOLUTION:

More generally let $V_n(r)$ denote the n -dimensional volume of the n -dimensional ball of radius r in \mathbb{R}^n . Then

$$\sum_{n=0}^{\infty} V_n(r) = e^{\pi r^2} \left(1 + \frac{2}{\sqrt{\pi}} \int_0^{r\sqrt{\pi}} e^{-t^2} dt \right).$$

This result appears (with proof) in the article “How small is the unit ball?”, *Math. Magazine* 62 (1989), 101-107.