

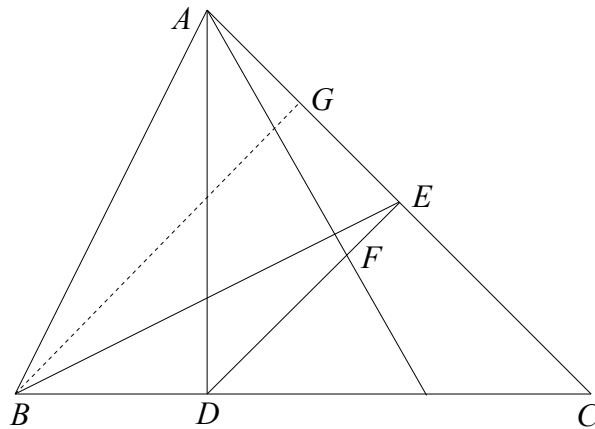
Configuration of Perpendiculars

1351. *Proposed by Florin S. Pîrvănescu, Slatina, Romania.*

In the acute triangle ABC , let D be the foot of the perpendicular from A to BC , let E be the foot of the perpendicular from D to AC , and let F be a point on the line segment DE . Prove that AF is perpendicular to BE if and only if $FE/FD = BD/DC$.

SOLUTION

Draw $BG \perp AC$



$BD/DC = GE/EC$. It is elementary that $\triangle ADC \sim \triangle BGC \sim \triangle AED$. In particular, sides of $\triangle BGC$ are perpendicular to corresponding sides of $\triangle AED$. In these triangles, AF and BE are cevians from corresponding vertices. Thus, $AF \perp BE$ if and only if $GE/EC = EF/FD$ and by substitution $AF \perp BE$ if and only if $BD/DC = EF/FD$.

Note: This problem is a generalization of problem 1199, The Mathematics Magazine, Vol. 58, No. 4, Sep 1985, p. 243.