APPENDIX III — COLLISION FREQUENCY OF THERMAL MUONIUM

AIII.A Derivation

Consider a point particle of mass m and mean thermal velocity $\bar{v}$, moving freely in a uniform distribution of N spherical particles of radius R. If one defines the number density to be N/V, where V is the total volume of the sample, the mean free path L is then written

$$L = V\left(\pi R^2 N\right)^{-1} \qquad \text{(AIII.1)}$$

By dividing the mean free path L by the mean thermal velocity $\bar{v}$, one obtains the average time $\bar{t}$ between collisions, namely

$$\bar{t} = \frac{L}{\bar{v}} = V\left(\pi R^2 \bar{v} N\right)^{-1} \qquad \text{(AIII.2)}$$

Taking the reciprocal of Equation AIII.2 then gives the collision frequency, and substituting the definition of the mean thermal velocity, one has

$$F(T) = \frac{1}{\bar{t}} = \frac{N}{V}\left(\pi R^2\right)\bar{v} = \frac{N}{V}\left(\pi R^2\right)\left[\frac{8kT}{\pi m}\right]^{1/2} \qquad \text{(AIII.3)}$$

where k is Boltzmann's constant and T is the temperature.

AIII.A.1 Low Density Limit

For low packing densities (neglecting the volume of the solid), the number density is simply given by the equation

$$\frac{N}{V} = \frac{\rho}{M} = \left(\frac{3}{4\pi R^3}\right)\frac{\rho}{\rho_o} \qquad \text{(AIII.4)}$$

where M is the mass of one grain (particle), ρ is the mass packing density (i.e., after compression) of the target particles and ρ_o is the mass density of the bulk material (for SiO_2; $\rho_o = 2.2$ g/cm^3). Thus in the low density limit, the collision frequency is

$$F(T) = \left(\frac{3}{R}\right) \frac{\rho}{\rho_o} \left[\frac{k}{2\pi m}\right]^{1/2} T^{1/2} \qquad \text{(AIII.5)}$$

This equation is, however, not correct if the volume of the solid (i.e., the volume of the N particles) is significant with respect to the total volume of the sample.

AIII.A.2 High Density Limit

In the high packing density limit, the volume of the solid is no longer negligible, so that one must redefine the number density to be the number of particles (grains) per unit "free volume" V_f, namely

$$V_f = \left(V - V_{solid}\right) = V - N\left(\frac{4}{3}\pi R^3\right) = V\left[1 - \frac{N}{V}\left(\frac{4}{3}\pi R^3\right)\right] \qquad \text{(AIII.6)}$$

By combining Equations AIII.4 and AIII.6, one obtains

$$\frac{N}{V_f} = \frac{3}{4\pi\ R^3} \left[\frac{\rho_o}{\rho} - 1\right]^{-1} \qquad \text{(AIII.7)}$$

Using this "corrected" number density, the collision frequency for the high density limit is

$$F(T) = \frac{3}{R} \left(\frac{k}{2\pi m}\right)^{1/2} \left[\frac{\rho_o}{\rho} - 1\right]^{-1} T^{1/2} \qquad \text{(AIII.8)}$$

Notice that for low densities, Equation AIII.8 reduces to the expression of Equation AIII.5.

Q.E.D.