J Fusion Energ (2008) 27:108–110
DOI 10.1007/s10894-007-9114-1

ORIGINAL PAPER

Stability of Plasma Configurations During Compression

Edward L. Ruden · James H. Hammer

Published online: 24 July 2007

Abstract Magnetized Target Fusion (MTF) efforts are based on calculations showing that the addition of a closed magnetic field relaxes the driver pressure and pulse width requirements for inertial confinement fusion by reducing thermal conductivity. Instabilities that result in convective bulk transport at the Alfven time scale are of particular concern since they are much faster than the implosion time. This paper focuses on the hard-core z-pinch and the field reversed configuration (FRC), two competing geometries presently being explored for MTF. Instabilities during compression may result from a violation of the Kadomstev stability criteria for the former, and increased angular velocity due to angular momentum conservation for the latter. Basic analytic considerations are addressed to provide a baseline for more detailed modeling.

Keywords Kadomtsev profile · Magnetized target fusion · Field reversed configuration

Hard-core z-pinch

We consider here the adiabatic ideal magnetohydrodynamic (MHD) response of a hard-core z-pinch with purely azimuthal magnetic field and inner radius a to an outer conductor of initial radius $r = b_0$ compressing to $r = b$. The ''0'' subscript will identify an initial profile property throughout, except for Q_0 and μ_0. The Kadomtsev criteria [1] for stability of all azimuthal modes $\{m\}$ for the compressed state are then used to determine stability

$$\begin{aligned} Q_0 &= \tfrac{-(6+5\beta)}{20}\tfrac{r}{p}\tfrac{dp}{dr} \le 1 \quad (m = 0) \\ Q_1 &= -\beta\tfrac{r}{p}\tfrac{dp}{dr} \le 1 \quad (m \ge 1) \end{aligned} \tag{1}$$

We focus on the special case of an initial ''Kadomtsev profile'', defined as one which has $Q_0 = 1$ everywhere. This profile is of special interest because a hard-core z-pinch can theoretically be made that relaxes roughly into one [2]. Given this profile for our initial state, we have the following initial (0) properties parameterized by β,

$$\begin{aligned} \frac{p}{p_a} &= \frac{\beta^{5/2}(4+5\beta_a)^{5/2}}{\beta_a^{5/2}(4+5\beta)^{5/2}} \qquad \frac{r}{a} = \frac{\beta_a^{3/4}(4+5\beta)^{1/4}}{\beta^{3/4}(4+5\beta_a)^{1/4}} \\ \frac{B}{B_a} &= \frac{\beta^{3/4}(4+5\beta_a)^{5/4}}{\beta_a^{3/4}(4+5\beta)^{5/4}} \quad Q_1 = \frac{20\beta(5\beta_a+4)^{5/2}}{(5\beta+6)(5\beta+4)^{5/2}} \end{aligned} \tag{2}$$

Here and elsewhere, subscripts ''a'' and ''b'' identify property values at $r = a$ and $r = b$, respectively. Q_1 falls off monotonically from its maximum at $r_0 = a$. The configuration is, furthermore, stable to $m \ge 1$, with $Q_{a1} \le 1$, if $\beta_a \le 2/5$.

The time invariant mass and magnetic flux per unit axial length within a fluid element of an ideal MHD pinch *initially* bounded by r_0 and $r_0 + dr_0$ during compression are $2\pi r\rho dr_0$ and Bdr_0, respectively. The ratio, then, is a time invariant of the fluid element (parameterized by r_0) called the Kadomtsev parameter $K = K(r_0)$. Expressing in terms of p instead of ρ via the adiabatic invariant $p\rho^{-5/3}$, we have

$$K(r_0) = \frac{p^{3/5}r}{B} \tag{3}$$

E. L. Ruden (✉)
Air Force Research Laboratory, Directed Energy Directorate, Kirtland AFB, NM, USA
e-mail: edward.ruden@kirtland.af.mil

J. H. Hammer
Lawrence Livermore National Laboratory, Livermore, CA, USA

One finds from substitution that $K(r_0)$ is uniform for a Kadomtsev profile so, when adiabatically compressed,

$$\frac{p^{3/5} r}{B} = \frac{p_{a0}^{3/5} a}{B_{a0}} \tag{4}$$

is a constant of both time and space. If we further assume that the plasma remains in equilibrium, Eq. 4 and the definition of β may be used to express the radial MHD pressure balance equation as a differential equation in p with reference to β, with the solution being *a Kadomtsev profile*. That is, the uniformity of $p^{3/5}r/B$ in equilibrium implies and is implied by (is equivalent to) a Kadomstev profile. Equations 2, therefore, remain valid during compression, and $m = 0$ remains marginally stable.

Solutions to β_a, B_a, p_a, Q_1, and other properties such as ρ and T needed to complete the description and stability analysis of the compressed state are found first by choosing a value of β_a and solving for those properties in terms of it. One finds from Eq. 4 solved at $r = a$ and the definition of β, for example,

$$\frac{B_a}{B_{a0}} = \left(\frac{\beta_{a0}}{\beta_a}\right)^3 \quad \frac{p_a}{p_{a0}} = \left(\frac{\beta_{a0}}{\beta_a}\right)^5 \tag{5}$$

From this one sees that compression, which causes B_a and p_a to rise, results in $\beta_a < \beta_{a0}$. Since $\beta_a \leq 2/5$ is the criterion from the stability of $m \geq 1$ for a Kadomstev profile, a configuration initially stable to these modes is stable under compression too. All modes, then, remain stable.

The value of b for the chosen β_a is found ex post facto. From the Kadomtsev r profile,

$$\frac{b}{a} = \frac{\beta_a^{3/4}(4 + 5\beta_b)^{1/4}}{\beta_b^{3/4}(4 + 5\beta_a)^{1/4}} \tag{6}$$

But for this we need β_b too. To this end, we derive a relationship between a given fluid element's β and its initial value β_0 from the fact that flux interior to that element is conserved,

$$\int_{\beta_a}^{\beta} B \frac{dr}{d\beta} d\beta = \int_{\beta_{a0}}^{\beta_0} B_0 \frac{dr_0}{d\beta_0} d\beta_0 \tag{7}$$

Plugging in the initial and compressed Kadomstev profiles (Eq. 2), and making use of Eq. 4 and definition of β, the result is

$$\frac{\beta_{a0}^3(5\beta_a + 4)}{\beta_a^3(5\beta_{a0} + 4)} \left(\frac{\ln\left(\frac{\beta(5\beta_a+4)}{\beta_a(5\beta+4)}\right) + 20(\beta - \beta_a)}{\ln\left(\frac{\beta_0(5\beta_{a0}+4)}{\beta_{a0}(5\beta_0+4)}\right) + 20(\beta_0 - \beta_{a0})} \right) = 1 \tag{8}$$

Solved numerically for the function $\beta(\beta_0)$ (given β_{a0} and β_a), we have $\beta_b = \beta(\beta_{b0})$ needed to find b, where β_{b0} is determined by numerically solving for it from the initial Kadomstev r profile evaluated at $r = b_0$,

$$\frac{b_0}{a} = \frac{\beta_{a0}^{3/4}(4 + 5\beta_{b0})^{1/4}}{\beta_{b0}^{3/4}(4 + 5\beta_{a0})^{1/4}} \tag{9}$$

Having $\beta(\beta_0)$ also allows one to calculate $\rho(\beta_0)$ and $T(\beta_0)$ from its initial profile via the adiabatic invariant,

$$\frac{\rho(\beta_0)}{\rho_0(\beta_0)} = \left(\frac{p(\beta(\beta_0))}{p_0(\beta_0)}\right)^{3/5} \quad \frac{T(\beta_0)}{T_0(\beta_0)} = \left(\frac{p(\beta(\beta_0))}{p_0(\beta_0)}\right)^{2/5} \tag{10}$$

Listed below are the results of a marginally stable Kadomtsev profile (β_{a0} = 2/5) initially with b_0/a = 10 compressed to the point where $\beta_a = 0.17$, subsequently determined to correspond to b/a = 1.2. This example was chosen to match the parameters of a published MHD simulation intended to represent this case [3].

$$\begin{array}{lll} \frac{b_0}{a} = 10 & \beta_{a0} = 0.4 & \beta_a = 0.17 \\ \frac{B_a}{B_{a0}} = 13.03 & \frac{B_b}{B_{b0}} = 76.77 & Q_{a1} = 0.496 \\ \frac{p_a}{p_{a0}} = 72.12 & \frac{p_b}{p_{b0}} = 4.733 \times 10^4 & \frac{b}{a} = 1.203 \\ \frac{\rho_a}{\rho_{a0}} = 13.03 & \frac{\rho_b}{\rho_{b0}} = 638.4 & \beta_{b0} = 1.633 \times 10^{-2} \\ \frac{T_a}{T_{ao}} = 5.536 & \frac{T_b}{T_{b0}} = 74.14 & \beta_b = 0.1311 \end{array} \tag{11}$$

The results are consistent with Fig. 5 of Ref. 3. In addition to the general properties already discussed, one noteworthy feature is the much greater compression and heating factor at larger radii.

In MHD simulations which include thermal conductivity, contact with the liner causes plasma pressure near the outer surface to drop, violating the Kadomtsev criterion locally. The resultant localized $m = 0$ instability can then convect impurities inward, at least a small distance. Also, the center conductor (which is not part of the Kadomtsev profile) can go $m = 0$ unstable, convecting impurities outward [2]. One question then is the radial extent of transport from the unstable walls. The effect of impurities can be significantly mitigated if the walls are coated with Li or, better yet, frozen D_2.

Field Reversed Configuration

Use of an FRC substantially resolves the wall contact issue, but at the cost of introducing a new (rotational) instability. An FRC has an open magnetic field outside a separatrix which effectively diverts wall material. However, FRC particles diffusing across the separatrix have a preferred angular momentum, causing the FRC within to counter-rotate in response [4]. End shorting of open magnetic fields lines is also thought to play a roll in spin-up [5]. When the FRC's rotational-diamagnetic flow frequency ratio α reaches a critical value of order unity, the FRC experiences rapid particle loss due to a rotational instability. This mode is conceptually similar to the Rayleigh–Taylor instability of a finite Larmour radius plasma [6], but with acceleration being centripetal [7].

The dynamics of the $n = 2$ mode is of particular concern for the MTF application because α theoretically increases significantly during wall compression by a conducting cylindrical liner. To show this, firstly, angular velocity Ω_R increases in proportion to R_s^{-2} from angular momentum conservation, where R_s is the separatrix radius. Meanwhile, $x_s \equiv R_s/R_c$ is conserved during cylindrical wall compression, where R_c is the liner inner radius (Tuszewski, p. 2058) [8]. Given this, plasma β is conserved (Tuszewski, Eq. 10) [8]. Given *this* and magnetic flux conservation, diamagnetic flow frequency Ω_{Di} is proportional to T_i, (Shimamura, Eq. 7 with $\Omega^* = -\Omega_{Di}$) [9]. The FRC's characteristic volume $V = \pi R_s^2 l_s$, meanwhile, decreases as R_s^N, where l_s is the separatrix length, and N is defined as the dimensionality of compression. Assuming adiabatic compression, $T_i V^{(5/3)-1}$ is conserved. Therefore, $\Omega_{Di} r_s^{2N/3} =$ constant, and Ω_{Di} increases in proportion to $R_s^{-2N/3}$. $\alpha = \Omega_R/\Omega_{Di}$, then, increases in proportion to $R_s^{-2}/R_s^{-2N/3} = R_s^{-2(1-N/3)}$. $N = 12/5$ for cylindrical wall compression (Tuszewski, Table V)[8], so α increases in proportion to $R_s^{-2/5}$ or, equivalently, $R_c^{-2/5}$. The (target) factor of 10 radial compression, then, increases α by a factor of $10^{2/5} \approx 2.5$.

Summary

The effect of compression on the stability of two leading candidates for MTF are presented. For a hard-core pinch, bulk stability is preserved, but radial convection of wall material is expected from the destabilization of an initial Kadomtsev profile near the wall. Further investigation is needed to assess its significance. Mitigation strategies such as using an overstabilized initial state and/or low Z wall loading are possible. For an FRC, disintegration during compression is expected due to unfavorable scaling of the rotational $n = 2$ instability's stabilitiy threshold parameter α. Mitigation techniques need further investigation.

References

1. B.B. Kadomtsev, in *Reviews of Plasma Physics*, vol. 2, ed. by M.A. Leontovich (Consultants Bureau, New York, 1966), pp. 153–199
2. V. Makhin, R.E. Siemon, B.S. Bauer, A. Esaulov, I.R. Lindemuth, V.I. Sotnikov, D.D. Ryutov, P.T. Sheehey, Phys. Plasmas **12**, 042312 (2005)
3. R.E. Siemon, W. Atchison, T. Awe, B. Bauer, A. Buyko, V. Chernyshev, T. Cowan, J. Degnan, S. Faehl, R.J. Fuelling, S. Garanin, T. Goodrich, A. Ivanovsky, I. Lindemuth, V. Makhin, V. Mokhov, R. Reinovsky, D. Ryutov, D. Scudder, T. Taylor, V. Yakubov, Nucl. Fusion **45**, 1148 (2005)
4. E.V. Belova, R.C. Davidson, H. Ji, M. Yamada, C.D. Cothran, M.R. Brown, M.J. Schaffer, Nucl. Fusion **46**, 162 (2006)
5. L.C. Steinhauer, Phys. Fluids **24**, 328 (1981)
6. K.V. Roberts, J.B. Taylor, Phys. Rev. Lett. **8**, 197 (1962)
7. E.L. Ruden, S. Zhang, T.P. Intrator, G.A. Wuden, Phys. Plasmas **13**, 122505 (2006)
8. M. Tuszewski, Nucl. Fusion **28**, 2033 (1988)
9. S. Shimamura, Y. Nogi, Fusion Technol. **9**, 69 (1986)