FRC rotation control using an electric field

Edward L. Ruden
Air Force Research Laboratory, Directed Energy Directorate

Michael H. Frese
NumerEx

A Field Reversed Configuration (FRC) spontaneously gains angular momentum about its z-axis over time until an instability with azimuthal mode number $n = 2$ develops. This is potentially a limiting factor for particle confinement in FRC's. Ions diffusing beyond the separatrix having a preferred angular momentum is one cause of rotation. The boundary conditions where the open magnetic field lines outside the separatrix exit the vacuum chamber resulting in a viscous torque being applied to the FRC at the separatrix is another. Controlling the axial electric field via equipotential conducting rings at a fused quartz tube's inner surface where the open field lines exit the vacuum to prevent rotation is considered. Torque on the FRC due to otherwise passive boundary conditions there may thereby be avoided, and spin-up due to particle diffusion countered. A steady state analysis of FRC rotation due to the boundary potential distribution of a perfectly conducting extended MHD plasma obeying generalized Ohm's law, in addition to numerical simulations of the process are presented.

FRC ions diffusing beyond the separatrix have a preferred angular momentum, causing the FRC within to counter-rotate in response[cite: Belova06]. Also, the boundary conditions where the open magnetic field lines outside the separatrix exit the vacuum chamber can result in a torque being applied to the FRC[cite: Steinhauer02].

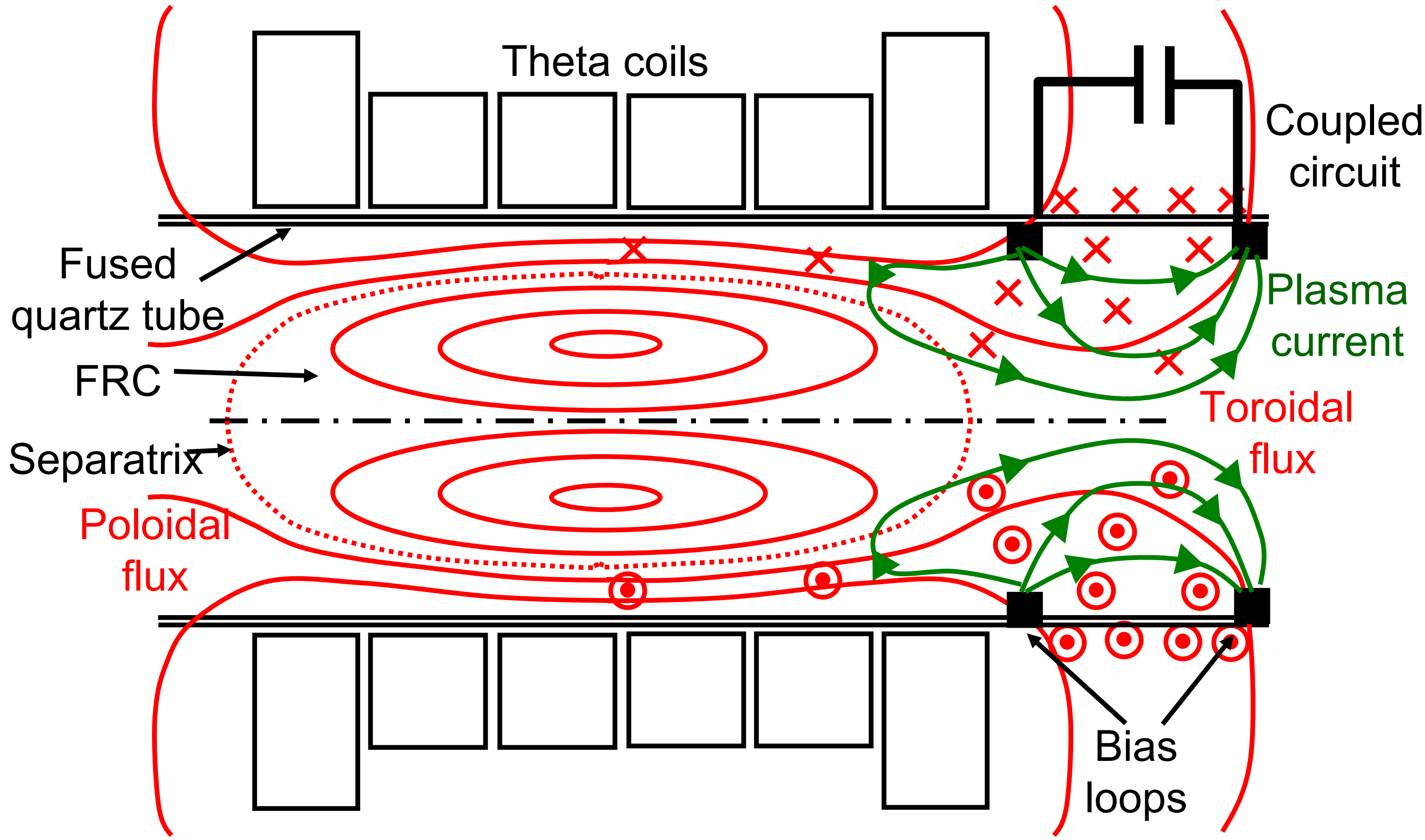
Belova06 E. V. Belova, R. C. Davidson, H. Ji, M. Yamada, C. D. Cothran, M. R. Brown, and M. J. Schaffer, Nucl. Fusion **46**, 162 (2006).

Steinhauer02 L. C. Steinhauer, Phys. Plasmas **9**, 3851 (2002).

Both mechanisms may contribute significantly to FRC rotation in the direction of the FRC azimuthal diamagnetic flow. When the FRC's rotational-diamagnetic flow frequency ratio α reaches a critical value of order unity, the FRC experiences rapid particle loss due to a rotational instability with azimuthal mode number $n = 2$.

This instability is observed for the FRC being studied for the Magnetized Target Fusion program [cite: Ruden06b], which employs an FRC inductively formed inside a long fused quartz tube. The idea of taking control over the electric field at the fused quartz tube inner surface beyond the formation coils via split conducting rings to control rotation is suggested. Torque on the FRC due to otherwise passive boundary conditions may thereby be avoided, and spin-up due to particle diffusion countered.

Ruden06b E. L. Ruden, Shouyin Zhang, T. P. Intrator, and G. A. Wuden, Phys. Plasmas **13**, 122505 (2006).



Qualitatively, current passing between the rings crosses the open magnetic field lines outside the separatrix to create azimuthal force, countertorquing the material exterior to the separatrix directly, and the interior FRC by viscous coupling. This current introduces toroidal flux.

Quantitatively, Steinhauer's steady state analysis [cite: Steinhauer02] of FRC rotation due to the boundary potential of a perfectly conducting extended MHD plasma (obeying generalized Ohm's law) can be used to estimate the optimal voltage.

Steinhauer02 L. C. Steinhauer, Phys. Plasmas **9**, 3851 (2002).

In steady state, the electric field can be represented by the negative gradient of a scalar potential ϕ, so the magnetically transverse ion velocity due to the combined effects of $\mathbf{E} \times \mathbf{B}$ and ion diamagnetic drifts may be written

$$\mathbf{u}_{\perp i} = \frac{1}{B^2}\left(\mathbf{B} \times \nabla\phi + \frac{1}{q_i n_i}\mathbf{B} \times \nabla p_i\right) \tag{1}$$

where centripetal acceleration has been neglected. Assuming symmetry about the FRC midplane, we then have at the midplane

$$u_{i\theta} = \frac{1}{B}\frac{d\phi}{dr} + \frac{1}{q_i n_i B}\frac{dp_i}{dr} \quad \text{(mid)} \tag{2}$$

The end-shorting mechanism for FRC spin-up is generally presented in the context of an electrically conducting boundary. However, from Eq. 1 we see that viscous drag on the plasma in the immediate vicinity of an insulating wall which reduces $\mathbf{u}_{\perp i}$ can have a significant impact in terms of lowering $\nabla\phi$ parallel to the wall. The $\nabla p_i/n_i$ term in Eq.1 often appears only as a relatively small correction to ideal MHD resulting from generalizing Ohm's law. However, thermal contact and impurity radiation near the wall may significantly reduce its magnitude, and therefore $\nabla\phi$, further. The insulator, then, could have a similar effect as a conductor in terms of end-shorting.

To determine ϕ here, generalized Ohm's law

$$\mathbf{E} = -(\mathbf{u}_i \times \mathbf{B} - \frac{1}{en_e}(\mathbf{J} \times \mathbf{B} + \nabla p_e)) + \eta \mathbf{J} \quad (3)$$

with resistivity $\eta = 0$ may be integrated long an open magnetic field line outside the separatrix from the vacuum wall boundary to the midplane. Neglecting variations in T_e along the field line (generally a reasonable assumption due to high thermal conductivity in that direction), we have,

$$\phi = \frac{kT_e}{e} \ln \frac{n_e}{n_{e0}(\psi)} + \phi_0(\psi) \quad \text{(mid)} \quad (4)$$

Here, ψ is the axial magnetic flux interior to r at the miplane,

$$\psi = 2\pi \int_{r_s}^{r} r' B(r') dr' \quad \text{(mid)} \quad (5)$$

r_s is the midplane separatrix radius, and $n_{e0}(\psi)$ and $\phi_0(\psi)$ are n_e and ϕ at the vacuum boundary surface interior to which the axial magnetic flux equals ψ, respectively.

Substituting into our expression for $u_{i\theta}$ (Eq. 2) and integrating from the r_s to variable midplane r, the boundary potential distribution that results in $u_{i\theta} = 0$ at the midplane is

$$\phi_0(\psi) = \frac{kT_e(r_s)}{e} \ln \frac{n_e(r_s)}{n_{e0}(0)} - \frac{kT_e(r)}{e} \ln \frac{n_e(r)}{n_{e0}(\psi)}$$

$$- \int_{r_s}^{r} \frac{1}{q_i n_i(r')} \frac{dp_i(r')}{dr'} dr' + \phi_0(0) \qquad \text{(mid)} \qquad (6)$$

If we now assume that $T = T_i = T_e$ are uniform, that the plasma is singly ionized, and that $n = n_i \approx n_e$, this expression simplifies to

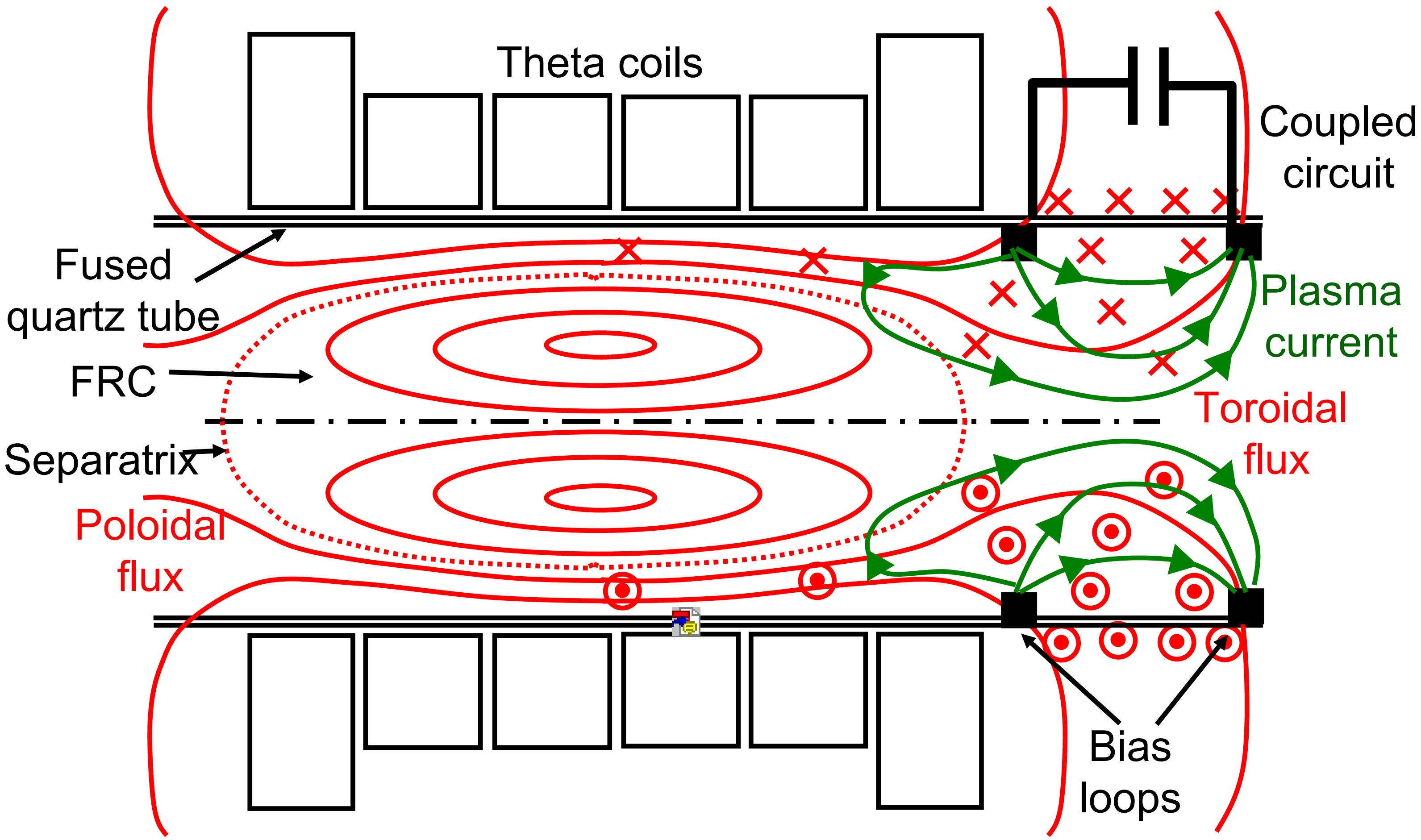
$$\phi_0(\psi) = \frac{kT}{e} \ln \frac{n_0(\psi) n^2(r_s)}{n_0(0) n^2(r)} + \phi_0(0) \quad \text{(mid)} \qquad (7)$$

Remember, n here is at the midplane and expressed as a function of r, while n_0 is at the insulator and expressed as a function of ψ.

A great deal of data on and analysis of FRX-L FRC’s is already available for substitution into Eq. 7 to determine the voltage requirements for the external bias supply required to neutralize rotation. Equation 7 indicates that, in volts, only a few times the plasma temperature in *electron* volts is required (~ 300 volts). The applied current must be sufficient to overcome the viscous drag from plasma wall contact. This current may be estimated, or at least a lower bound established, from the power required to torque up the FRC to its observed angular velocity in the observed time frame for instability onset, given the present passive boundary condtions.

Given the deuterium FRC's angular velocity of 2×10^6 s^{-1}, rms radius of 0.02 m, particle number per unit axial length 5×10^{19} m^{-1} and length of 0.2 m, the FRC's rotational kinetic energy is about 30 J. Controlling its rotation on a 10 μs time-scale, therefore, requires ~ 3 MW of power (~ 10 kA, a very modest capacitor circuit).

MACH2 simulations are planned to refine the theoretical description in terms of optimal conducting ring placement and bias interior to the fused quartz tube. The ring, in practice, must be either split or at least resistive enough to permit poloidal flux to pass between them.



MACH2 defines boundary conditions for $\mathbf{B}$ equivalent to a given total current I between the rings via Ampere's Law. Voltage V is then calculated by integrating $\mathbf{E}$ based on Generalized Ohm's Law. The current is dynamically adjusted for consistency with an external circuit.