Rayleigh–Taylor Instability With a Sheared Flow Boundary Layer

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***Abstract*—S. Chandrasekhar, in his book, *Hydrodynamic and Hydromagnetic Stability* (New York: Dover, 1961), derives the stability criteria for a semi-infinite uniform density incompressible inviscid fluid with uniform horizontal velocity supported in a gravitational field by one of higher density and opposite velocity. A transitional layer of inviscid fluid with a density equal to the average of the upper and lower fluids, and a horizontal velocity that varies linearly with depth from that of the upper fluid at the top to that of the lower fluid at the bottom is assumed. This analysis of the *Kevin–Helmholtz* (K–H) *instability* may be transformed into a model of the effect of such a velocity sheared boundary layer on the Rayleigh–Taylor (R–T) instability of modes with wave numbers in the direction of the sheared velocity by reversing the sign of the top–bottom density differential. Orthogonal modes are unaffected by the shear in the linear limit and are, therefore, R–T unstable unless an independent mechanism for their stabilization is present, such as a magnetic field orthogonal to the sheared velocity. The combined R–T/K–H stability analysis is, therefore, expected to be most applicable for magnetically accelerated media such as a Z pinch with an axial velocity sheared outer layer orthogonal to the outer azimuthal magnetic field which drives the implosion.**

***Index Terms*—Hydrodynamic instability, Kevin–Helmholtz, Rayleigh–Taylor, sheared flow, Z pinch.**

I. Introduction

THE GEOMETRY of interest is shown in Fig. 1. We have, in equilibrium

$$\begin{aligned} \rho=\rho_+=\rho_0(1+\varepsilon) \quad &\text{and} \quad V=+V_0 \quad \text{for} \quad z>+d\\ \rho=\rho_0 \quad &\text{and} \quad V=V_0 z/d \quad \text{for} \quad +d>z>-d\\ \rho=\rho_-=\rho_0(1-\varepsilon) \quad &\text{and} \quad V=-V_0 \quad \text{for} \quad z<-d \end{aligned} \tag{1}$$

where ρ is the piece-wise uniform density, $v_x = V$ is the equilibrium fluid velocity in the horizontal (x) direction, and the gravitational field is $-g\mathbf{z}$. The dispersion relation for linear perturbations of this geometry, and stability diagrams for the cases of $\varepsilon \ll 1$ and $\varepsilon = 1$ were presented by the author at Sandia National Laboratory's 1996 Rayleigh–Taylor Workshop (unpublished). The dispersion relation was later generalized by an attendee of the Workshop (N. F. Roderick) for arbitrary constant values of (in our notation) ρ_+, ρ_0, and ρ_-. This general form was published with stability and unstable growth rate diagrams for the case of $\rho_- = 0$ and $\rho_+ = \rho_0$, and for the case of $\rho_- = \rho_+ = 0$ [1]. The present author concluded at the time that this publication adequately represents the gravitational inversion of Chandrasekhar's treatment of the Kevin–Helmholtz (K–H) instability [2], so no effort was made to publish the Workshop calculations until now. Nonetheless, the originally treated case better represents an axial velocity sheared layer localized to a relatively thin boundary layer on the outside of a Z pinch, where magnetic diffusion causes a continuous transition in plasma density from vacuum to bulk. This is significant because it is found in *all* cases studied that only modes within a range of wave numbers are stabilized by velocity shear, below and above which modes are still unstable. Modes with larger wave numbers, though, generally grow more rapidly if unstable and are, therefore, of greater concern. Such modes are localized to the outer layers of the pinch column in their linear phase. Shearing the flow primarily in this layer may stabilize modes up to a high wave number more efficiently. Such efficiency is important since the kinetic energy density associated with the sheared flow must be the same order of magnitude as the implosion itself to be effective. These considerations, current investigations at University of Nevada, Reno, on creating such a sheared flow boundary layer, and requests for a citable reference for the Workshop calculations lead to a reassessment of the need to publish the calculations. An additional consideration is that the cases solved and methodology used here are more closely analogous to Chandrasekar's original treatment, but with the direction of gravity reversed. Therefore, an educational comparison to Chandrasekar's seminal textbook problem may be made.

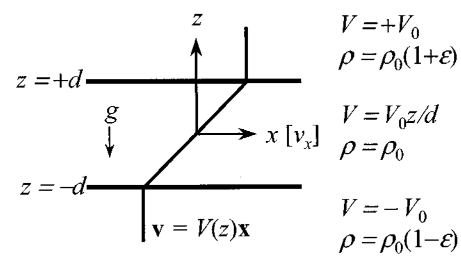

Fig. 1. Equilibrium configuration for the presented R–T instability model of a horizontally translating inviscid incompressible fluid supported by a lighter one of opposite velocity via an intermediate boundary layer of intermediate density and with a sheared fluid velocity.

It is anticipated that velocity shear stabilization will find its greatest use where other mechanisms help stabilize modes orthogonal to the sheared flow and/or with wave numbers outside the shear stabilized zone. For example, a magnetic field orthogonal to the sheared flow stabilizes the former [3]. And, magnetic diffusion at shorter wavelengths [4] (at least for highly compressible media [5]) and/or magnetic shear at longer wavelengths [6] can at least mitigate the latter.

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Publisher Item Identifier S 0093-3813(02)05324-9.

II. Dynamic Equations and Boundary Conditions

We assume the equation of motion of an incompressible inviscid fluid with isotropic pressure p, velocity vector $\mathbf{v}$, and density ρ in a uniform gravitational field of magnitude g acting in the $-\mathbf{z}$ direction ($\mathbf{x}$ and $\mathbf{z}$ are the x and z directional unit vectors, respectively)

$$\rho \frac{\partial}{\partial t} \mathbf{v} + \rho(\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = -\boldsymbol{\nabla} p - g\rho\mathbf{z}. \tag{2}$$

Incompressibility implies

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0. \tag{3}$$

Discontinuities in density across surfaces are allowed in our problem. The boundary conditions at such surfaces are that the surface normal component of the fluid velocity and the pressure are continuous across the surface

$$\mathbf{v_1}\cdot\mathbf{n} = \mathbf{v_2}\cdot\mathbf{n} \tag{4}$$

$$p_1 = p_2. \tag{5}$$

Subscripts "1" and "2" refer to values immediately above and below the boundary, respectively, and $\mathbf{n}$ is the surface normal unit vector.

III. General Solution

We may rewrite (2) and (3) in a form valid exclusively *within* each of the three density regions in terms of equilibrium terms plus first order two dimensional corrections $u \equiv \delta v_x$, $w \equiv \delta v_z$, and δp due to upper and lower boundary perturbations. $\delta\rho = 0$ within each region by assumption. Harmonic dependences of the form $\exp i(kx + \omega t)$ are assumed, where $k > 0$ by convention, and second order terms are dropped. The x and z components of (2) within each of the three regions are then, respectively

$$i\rho\omega u + \rho w \frac{\partial}{\partial z} V + \rho V iku = -ik\delta p \tag{6}$$

$$i\rho\omega w + \rho V ikw = -\frac{\partial}{\partial z} \delta p. \tag{7}$$

Equation (3) becomes,

$$iku + \frac{\partial}{\partial z} w = 0. \tag{8}$$

u may be eliminated between (6) and (8) by solving the latter for u and substituting into the former

$$i\rho(\omega + kV)\frac{\partial}{\partial z} w - i\rho kw \frac{\partial}{\partial z} V = -k^2 \delta p. \tag{9}$$

δp may be eliminated between (7) and (9) by multiplying the former by k^2, differentiating the later with respect to z, and equating the left-hand sides

$$\frac{\partial^2}{\partial z^2} w - \left[\frac{k \frac{\partial^2}{\partial z^2} V}{(\omega + kV)} + k^2\right] w = 0. \tag{10}$$

Equation (10) is valid for a constant density region with arbitrary $V(z)$. It is derived in a more general form elsewhere [7]. $\partial^2 V/\partial z^2 = 0$ within each of the three regions of interest in (1), so (10) reduces to

$$\frac{\partial^2}{\partial z^2} w - k^2 w = 0. \tag{11}$$

The solution which approaches zero as $z \to \pm\infty$ is

$$\begin{aligned} w_+ &= A_+ e^{-kz}, && \text{for} \quad z > +d \\ w_0 &= A_0 e^{-kz} + B_0 e^{+kz}, && \text{for} \quad +d > z > -d \\ w_- &= A_- e^{+kz}, && \text{for} \quad z < -d. \end{aligned} \tag{12}$$

IV. Imposing Boundary Conditions

The perturbed value of a property $Q(x, z, t)$ just inside the boundary of a given density region is, to first order

$$\begin{aligned} Q &= Q_{eq}(x, \pm d + \delta z_\pm, t) + \delta Q(x, \pm d, t) \\ &= Q_{eq}(x, \pm d, t) + \left[\frac{\partial Q_{eq}}{\partial z}\right]_{z=\pm d} \delta z_\pm + \delta Q(x, \pm d, t) \end{aligned} \tag{13}$$

where Q_{eq} is the equilibrium distribution valid within the region and $\delta z_\pm$ are the axial perturbations of the upper or lower boundaries as functions of x and t. Subscripts "$+$" and "$-$" in this section refer to the upper and lower boundaries or regions (depending on context), respectively. $\delta z_\pm$ may be expressed in terms of the value of w at the boundaries by equating the later to the convective time derivative of the former

$$\begin{aligned} w(x, \pm d, t) &= \frac{d}{dt} \delta z_\pm(x, t) \\ &= \left[\frac{\partial}{\partial t} \delta z_\pm + (\mathbf{v} \cdot \boldsymbol{\nabla})\delta z_\pm\right]_{z=\pm d} \\ &= i(\omega \pm kV_0)\delta z_\pm. \end{aligned} \tag{14}$$

We first take $Q = \mathbf{v} \cdot \mathbf{n}$ in (13) and satisfy (4). The resultant $[(\partial \mathbf{v_{eq}}/\partial \mathbf{z})\boldsymbol{\delta z}_\pm \cdot \mathbf{n}_\pm]$ term is second order and, therefore, not needed since $(\partial \mathbf{v_{eq}}/\partial \mathbf{z})\boldsymbol{\delta z}_\pm$ only has an x component, and the unit normal vectors $\mathbf{n}_\pm$ only have a first order x correction. We, therefore, need only ensure the continuity of $\delta\mathbf{v} \cdot \mathbf{n}_\pm$ to satisfy (4). Furthermore, since $\delta\mathbf{v}$ is only a first order term, only the zeroth order contribution to $\mathbf{n}_\pm$ (i.e., $\mathbf{z}$) is needed to satisfy (4) to first order. This means $w_\pm = w_0$ at $z = \pm d$. From (12), then

$$\begin{aligned} A_+ e^{-kd} &= A_0 e^{-kd} + B_0 e^{+kd} \\ A_- e^{-kd} &= A_0 e^{+kd} + B_0 e^{-kd}. \end{aligned} \tag{15}$$

We now take $Q = p$ in (13) and satisfy (5). Using $\partial p_{eq}/\partial z = -\rho g$ (where ρ is the regionally constant density) and δp as solved from (9) at the boundaries of all three regions, (5), (13), and (14) imply

$$\begin{aligned} (1+\varepsilon)\left[\frac{gw_0}{\omega + kV_0} + \frac{\omega + kV_0}{k^2}\frac{\partial w_+}{\partial z}\right]_{z=+d} &= \left[\frac{gw_0}{\omega + kV_0} + \frac{\omega + kV_0}{k^2}\frac{\partial w_0}{\partial z} - \frac{w_0 V_0}{kd}\right]_{z=+d} \\ \left[\frac{gw_0}{\omega - kV_0} + \frac{\omega - kV_0}{k^2}\frac{\partial w_0}{\partial z} - \frac{w_0 V_0}{kd}\right]_{z=-d} &= (1-\varepsilon)\left[\frac{gw_0}{\omega - kV_0} + \frac{\omega - kV_0}{k^2}\frac{\partial w_-}{\partial z}\right]_{z=-d}. \end{aligned} \tag{16}$$

Substituting in the general solutions from (12), expressing coefficients $A_\pm$ in terms of A_0 and B_0 using (15), and rearranging (16) yield

$$-\frac{A_0}{B_0}\,e^{-\kappa}=1-\frac{\kappa(\nu+1)^2}{(\nu+1)-\frac{1}{2}\,\varepsilon\kappa(\nu+1)^2-J}$$
$$-\frac{B_0}{A_0}\,e^{-\kappa}=1-\frac{\kappa(\nu-1)^2}{-(\nu-1)+\frac{1}{2}\,\varepsilon\kappa(\nu-1)^2-J} \quad (17)$$

where we define dimensionless variables

$$\nu=\frac{\omega}{kV_0},\quad \kappa=2kd,\quad \text{and}\quad J=\frac{\varepsilon g d}{V_0^2}. \quad (18)$$

J is the negative of the Richardson number used in Chandrasekhar's Kevin–Helmholtz instability analysis. Equations (17) may also be derived as a special case of Shumlak and Roderick's [1] (18) and (16), respectively, by replacing ρ_a, ρ_b, and ρ_c, in the latter with $\rho_0(1-\varepsilon)$, ρ_0, and $\rho_0(1+\varepsilon)$, respectively, as defined here. To avoid confusion, note that this reference uses a different definition for ε in their subsequent analysis. Multiplying (17) together, one obtains the dispersion relation for the geometry

$$e^{-2\kappa}=\left[1-\frac{\kappa(\nu+1)^2}{(\nu+1)-\frac{1}{2}\,\varepsilon\kappa(\nu+1)^2-J}\right]\times\left[1-\frac{\kappa(\nu-1)^2}{(\nu-1)+\frac{1}{2}\,\varepsilon\kappa(\nu-1)^2-J}\right]. \quad (19)$$

This expression is identical to Chandrasekhar's dispersion relation for the Kevin–Helmholtz instability [2, Ch. XI, eq. (53)], except that the sign of ε and J have been reversed. This significantly changes the stability analysis and criteria, as demonstrated in the two cases presented below.

V. Case I: Small ε Limit

Equation (19) is a quartic in ν^2 when expanded. We obtain a more tractable quadratic in ν^2, though, if we assume that ε is small, but that J is still significant (due to a small V_0 or large g, for example)

$$0=\kappa^2\nu^4-\left(1-e^{-2\kappa}+2\kappa^2-2\kappa(J+1)\right)\nu^2 +\left[(1-J)(1-e^{-\kappa})-\kappa\right]\left[(1-J)(1+e^{-\kappa})-\kappa\right]. \quad (20)$$

This expression is useful for cases where the relative change in density is small.

The solutions for ν^2 are, from the quadratic formula

$$\nu_\pm^2=\frac{\left(1-e^{-2\kappa}+2\kappa^2-2\kappa(J+1)\right)\pm\sqrt{\Delta}}{2\kappa^2} \quad (21)$$

where

$$\Delta=\left(1-e^{-2\kappa}+2\kappa^2-2\kappa(J+1)\right)^2 -4\kappa^2\left[(1-J)(1-e^{-\kappa})-\kappa\right]\left[(1-J)(1+e^{-\kappa})-\kappa\right]. \quad (22)$$

ν^2 must be a positive real number for a given mode to be stable. Therefore, a necessary and sufficient condition that all modes for a given J and κ be stable are that all the following criteria are met:

$$\begin{array}{ll}\text{I} & \Delta\geqslant 0\\ \text{II} & \nu_+^2+\nu_-^2>0\\ \text{III} & \nu_+^2\nu_-^2>0.\end{array} \quad (23)$$

To see why, consider the following. Criterion I implies from (21) that both ν^2 solutions are real. Given this, Criterion II implies that at least one of the real solutions is positive. Given *these*, Criterion III implies that both must be positive. Furthermore, if any one of the criteria is not met, then at least one solution must be nonreal or negative.

To determine specific J–κ relationships required for stability, the following representation of Δ is useful. Manipulations of (22) yield

$$\Delta=\left[2\kappa(1-J)e^{-\kappa}+e^{\kappa}f\right]^2-f\left[(2\kappa-1)e^{\kappa}+e^{-\kappa}\right]^2 \quad (24)$$

where

$$f=(2\kappa-1)^2-e^{-2\kappa}. \quad (25)$$

Criterion I is seen from (24) to be equivalent to

$$\left[2\kappa(1-J)e^{-\kappa}+e^{\kappa}f\right]>|f|^{1/2}\left|(2\kappa-1)e^{\kappa}+e^{-\kappa}\right|$$

or

$$\left[2\kappa(1-J)e^{-\kappa}+e^{\kappa}f\right]<-|f|^{1/2}\left|(2\kappa-1)e^{\kappa}+e^{-\kappa}\right| \quad (26)$$

where "$||$" means absolute value. Solving for J, this becomes

$$J<1-\frac{|f|^{1/2}g-e^{\kappa}f}{2\kappa e^{-\kappa}}\quad\text{or}\quad J>1+\frac{|f|^{1/2}g-e^{\kappa}f}{2\kappa e^{-\kappa}} \quad (27)$$

where

$$g=\left|(2\kappa-1)e^{\kappa}+e^{-\kappa}\right|. \quad (28)$$

Criterion II, meanwhile, is seen from (21) to be equivalent to

$$1-e^{-2\kappa}+2\kappa^2-2\kappa(J+1)>0. \quad (29)$$

Solving for J, Criterion II becomes

$$J<\frac{1-e^{-2\kappa}+2\kappa^2-2\kappa}{2\kappa}. \quad (30)$$

Noting from the quadratic formula that the product of the two roots of quadratic $0=ax^2+bx+c$ is c/a, we see from (20) that

$$\nu_+^2\nu_-^2=\frac{\left[(1-J)(1-e^{-\kappa})-\kappa\right]\left[(1-J)(1+e^{-\kappa})-\kappa\right]}{\kappa^2}. \quad (31)$$

Criterion III, then, is equivalent to

$$\left[(1-J)(1-e^{-\kappa})-\kappa\right]>0\ \text{and}\ \left[(1-J)(1+e^{-\kappa})-\kappa\right]>0$$

or

$$\left[(1-J)(1-e^{-\kappa})-\kappa\right]<0\ \text{and}\ \left[(1-J)(1+e^{-\kappa})-\kappa\right]<0. \quad (32)$$

This may be rewritten as

$$1-J>\frac{\kappa}{1-e^{-\kappa}}\quad\text{and}\quad 1-J>\frac{\kappa}{1+e^{-\kappa}}$$

or

$$1-J<\frac{\kappa}{1-e^{-\kappa}}\quad\text{and}\quad 1-J<\frac{\kappa}{1+e^{-\kappa}}. \quad (33)$$

Since all the numerators and denominators in (33) are positive and, of the denominators, $1+e^{-\kappa} > 1-e^{-\kappa}$ for all positive κ, the second inequality of the first line and the first inequality of the second line are superfluous. Criterion III may, therefore, be written (after solving for J) as

$$J < 1 - \frac{\kappa}{1-e^{-\kappa}} \quad \text{or} \quad J > 1 - \frac{\kappa}{1+e^{-\kappa}}. \tag{34}$$

The stability criteria may be summarized from (27), (30), and (34) as

$$(J < J_1 \text{ or } J > J_2) \text{ and } (J < J_3) \text{ and } (J < J_4 \text{ or } J > J_5) \tag{35}$$

where

$$\begin{aligned} J_1 &= 1 - \frac{|f|^{1/2}g - e^{\kappa}f}{2\kappa e^{-\kappa}} \qquad J_2 = 1 + \frac{|f|^{1/2}g - e^{\kappa}f}{2\kappa e^{-\kappa}} \\ J_3 &= \frac{1-e^{-2\kappa}+2\kappa^2-2\kappa}{2\kappa} \\ J_4 &= 1 - \frac{\kappa}{1-e^{-\kappa}} \qquad J_5 = 1 - \frac{\kappa}{1+e^{-\kappa}} \\ f &= (2\kappa-1)^2 - e^{-2\kappa} \qquad g = |(2\kappa-1)e^{\kappa} + e^{-\kappa}|. \end{aligned} \tag{36}$$

Upon inspection of the plots of the above, some redundancy is present and the stability criteria may be shortened to

$$J < J_4 \text{ or } (J < J_1 \text{ and } J > J_5 \text{ and } \kappa > 0.9). \tag{37}$$

J_4 is always negative, so $J < J_4$ only applies for negative J. This corresponds to the Kevin–Helmholtz instability analysis of a light fluid supported by a heavy fluid covered by Chandrasekhar [2]. The stability boundaries for positive J are illustrated in Fig. 2, along with those for Case II ($\varepsilon = 1$), discussed next.

VI. Case II, $\varepsilon = 1$

One case of interest for magnetically accelerated media is $\varepsilon = 1$. This corresponds to a zero density (magnetic) fluid accelerating a fluid of density $2\rho_0$ with a transitional region of density ρ_0. Expanding (19) in this case gives

$$\begin{aligned} 0 = &\left(-3\kappa^2 - e^{-2\kappa}\kappa^2\right)\nu^4 + \left(4e^{-2\kappa}\kappa - 4\kappa\right)\nu^3 \\ &+ \left(-8\kappa + 4 - 8J\kappa + 6\kappa^2 + 2e^{-2\kappa}\kappa^2 - 4e^{-2\kappa}\right)\nu^2 \\ &+ \left(-8J\kappa + 8e^{-2\kappa}J\kappa + 4\kappa - 4e^{-2\kappa}\kappa\right)\nu \\ &- 4 + 8J + 8\kappa - 4J^2 - 8e^{-2\kappa}J + 4e^{-2\kappa} \\ &+ 4e^{-2\kappa}J^2 - 3\kappa^2 - e^{-2\kappa}\kappa^2 - 8J\kappa. \end{aligned} \tag{38}$$

The four roots of this quartic may be determined numerically for any give numerical values of κ and J. Any nonreal root for a point in (κ, J) space implies instability. Sampling select values of κ over a range of J, the upper and lower bounds for which all modes are stable are designated by "+" and "$*$," respectively, in Fig. 2.

VII. Conclusion

An alternate expression for J is

$$J = \frac{g}{\rho_0}\frac{\triangle\rho/\triangle z}{(\triangle v_x/\triangle z)^2} \tag{39}$$

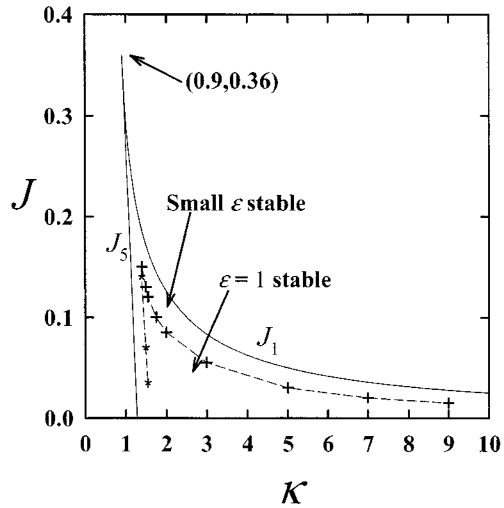

Fig. 2. Stability boundaries in (κ, J) space, where $J = \varepsilon g d/V_0^2$ is the negative Richardson number, and $\kappa = 2kd$ is the normalized wave number of the perturbation. The geometry of Fig. 1 is R–T stable for modes in the direction of sheared flow between the solid curves J_5 and J_1 for a small relative density differential ($\varepsilon \ll 1$), and between the dashed curves marked by "$*$" and "+" for $\varepsilon = 1$.

where the $\triangle$ terms are the changes in the corresponding values over the sheared flow layer. J is seen in this form to be proportional to the ratio of the mean density gradient over the region to the square of the velocity shear. Equation (39) is a useful representation for estimating an effective value of J when applying the simplified model presented here to more realistic configurations.

The small ε curve of Fig. 2 is at least of academic interest as it provides a direct extension of Chandrasekhar's solution [2, Fig. 119] for negative J (g), as he defines it (the negative of our J). This case may also find use in the study of flow shear effects at the boundary of atmospheric and/or ocean layers of different temperature. The $\varepsilon = 1$ curve is of greater interest for Z pinches and other magnetically accelerated media in cases where the magnetic skin depth is smaller than the imploding plasma shell thickness. If the magnetic skin depth coincides with the sheared layer thickness $2d$, for example, R–T wavelengths smaller than this (i.e., $\kappa > 2\pi$) do not grow significantly due to magnetic diffusion dissipation [4]. We see from Fig. 2 that for $J < 0.025$, only modes with $\kappa < 1.5$ should then be a problem. This corresponds to wavelengths over four times the skin depth. Such modes are generally of lesser concern due to their relatively slow growth rates and small initial amplitudes in, for example, wire array implosions, where axial uniformity is good on this scale. An axial magnetic field frozen into the plasma bulk, and the resulting magnetic shear, can also have a stabilizing effect on wavelengths much longer than the diffusion depth.

References

[1] U. Shumlak and N. F. Roderick, "Mitigation of the Rayleigh–Taylor instability by sheared axial flows," *Phys. Plasmas*, vol. 5, pp. 2384–2389, 1998.
[2] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*. New York: Dover, 1961.

[3] G. Schmidt, *Physics of High Temperature Plasmas*, 2nd ed. New York: Academic, 1979.
[4] N. F. Roderick and T. W. Hussey, "A model for the saturation of the hydrodynamic Rayleigh–Taylor instability," *J. Appl. Phys.*, vol. 56, pp. 1387–1390, 1984.
[5] E. L. Ruden and D. E. Bell, "Rayleigh–Taylor stability criteria for elastic-plastic solid plates and shells," *J. Appl. Phys.*, vol. 28, pp. 163–170, 1997.
[6] W. M. Manheimer and C. N. Lashmore-Davies, *MHD and Microinstabilities in Confined Plasma*. New York: IOP, 1989.
[7] G. I. Taylor, "Effect of variation in density on the stability of superposed streams of fluid," *Proc. Roy. Soc. London, A*, vol. 132, pp. 499–523, 1931.

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