DYNAMIC DEFORMATION OF A SOLENOID WIRE DUE TO INTERNAL MAGNETIC PRESSURE, REVISED

E. L. Ruden[ξ], G. F. Kiuttu
Air Force Research Laboratory, Directed Energy Directorate
AFRL/DEHP, 3550 Aberdeen Ave. SE
Kirtland AFB, NM 87117-5776, USA

M. H. Frese, S. D. Frese
Numerex, 2309 Renard Place SE
Albuquerque, NM 87106 USA

Abstract

Deformation of the wire used in the windings of an inertially confined (single use) solenoid used to produce a pulsed high magnetic field is potentially the limiting factor for the magnitude and duration of the magnetic field produced. The rising magnetic pressure at the wire surface can become large enough to cause the cross section of the wire to plastically deform on a time scale shorter than the overall solenoid disassembly time. This may result in short circuiting due to insulator breakage and/or physical contact of adjacent windings. An analytic approximation modeling the deformation dynamics is presented which takes into account both inertial and material yield strength effects. The model is validated by comparison to two dimensional magnetohydrodynamic simulations of the process by Numerex's MS Windows version of AFRL's MACH2. Cases ranging from those where yield strength has a negligible effect on the deformation to where yield strength is significant are considered. This paper expands on work presented at the previous IEEE IPPC[1].

I. INTRODUCTION

Consider a long solenoid wound from round wire made of metal with a given yield strength and having a diameter small compared to the solenoid diameter. We wish to model the deformation magnitude of the wire cross section due to an internal magnetic pressure high enough to cause rapid disassembly (explosion). A detailed model of the elastic-plastic flow requires a two-dimensional magnetohydrodynamic simulation with material strength treatment. A simpler analytic model, however, is desirable to parameterize scaling relationships, permit more timely design guidance, and provide physical insight into the deformation process. For the latter, we represent the wire in planar geometry by a square one of the same cross sectional area that, by assumption, remains rectangular during deformation. The solenoid radius is assumed to be large enough to permit hoop stress effects to be neglected during the time of interest. To permit an analytic solution, the magnetic pressure is represented by a uniform pressure applied to one side of the square that rises instantaneously to a constant value P at time t = 0. A free standing wire and one bounded by a rigid wall opposite the pressurized side are considered. These are intended to represent a free standing solenoid and one reinforced by a surrounding tube, respectively. The dynamic width of the wire is calculated assuming conservation of global energy and momentum.

The relationship between P used in the model and the central solenoid magnetic field B_1 must take into account the fact that the local magnetic field at the wire surface is higher than B_1. To determine an effective P, we equate the initial pdV work performed per winding of an ideal solenoid by expanding its radius by a small amount dR to the work performed on our square wire by pressure P over the same displacement,

$$\frac{B_1^2}{2\mu_0} \times 2\pi R_0 S dR = P \times 2\pi R_0 A_0 dR \qquad (1)$$

Here, R_0 is the initial solenoid radius, S is the wire spacing, r_0 is the initial wire radius, A_0 is the initial side length of a square wire of the same area, and $\mu_0 = 4\pi \times 10^{-7}$ H-m^{-1}. From this,

$$P = \frac{S}{A_0}\frac{B_1^2}{2\mu_0} \quad \text{where} \quad A_0 = \sqrt{\pi} r_0 \qquad (2)$$

In regard to the aforementioned detailed treatment, the analytic models are checked against the code MACH2[2][3] for a range of conditions to establish their suitability for providing general design criteria for similar geometries.

[ξ] email: ruden@plk.af.mil

II. DEFORMATION OF A RECTANGULAR WIRE DUE TO PRESSURE ON ONE SIDE

Referring to "top" and "bottom" as illustrated in Fig.1, we first calculate the deformation of a free rectangular wire (for generality) due to a uniform and constant pressure P on the top surface, assuming the wire remains rectangular. We define our (x, y) coordinates to be in the accelerated reference frame of the wire, with the origin at the center of mass. The dynamic wire width is A (A_0 initially), the height is B (B_0 initially), and the center of mass speed is V. Uniaxial yield strength Y and density ρ are assumed constant. We further assume the simplest possible expression for the material velocity field $\mathbf{v}$ in the center of mass,

$$v_x = \frac{dA}{dt}\frac{x}{A} \qquad v_y = \frac{dB}{dt}\frac{y}{B} \qquad v_z = 0 \tag{3}$$

From incompressibility,

$$AB = A_0 B_0 \tag{4}$$

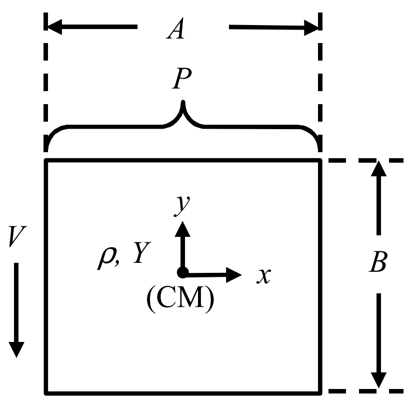

Figure 1. Geometry for analytic model of free standing solenoid wire deformation.

The Levy-Mises equation for rigid plastic flow[4] is assumed,

$$\mathbf{S} = \sqrt{\frac{2}{3}} Y \mathbf{D} / \sqrt{\mathbf{D}\cdot\mathbf{D}} \tag{5}$$

where $\mathbf{S}$ is the deviatoric stress tensor ($\mathbf{S}\cdot\mathbf{I} = 0$), and $\mathbf{D}$ is the deviatoric strain rate tensor,

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v} + \nabla\mathbf{v}^T) - \frac{1}{3}(\nabla\cdot\mathbf{v})\,\mathbf{I} \tag{6}$$

Incompressibility implies $\nabla\cdot\mathbf{v} = 0$. Substituting Eq. 3 into Eq. 6 then gives, for the nonzero $\mathbf{D}$ elements,

$$D_{xx} = -\frac{1}{A}\frac{dA}{dt} \qquad D_{yy} = \frac{1}{B}\frac{dB}{dt} \tag{7}$$

Since $\mathbf{S}$ is proportional $\mathbf{D}$ in this model, our neglect of hoop stress is equivalent to neglecting D_{zz} due to solenoid expansion.

The time derivatives of the external work, plastic work[4], and kinetic energy per unit length of wire are, respectively,

$$\frac{dW}{dt} = PA\left(V - \frac{1}{2}\frac{dB}{dt}\right) \qquad \frac{dE_p}{dt} = \int_{\text{Area}} (\mathbf{S}\cdot\mathbf{D})\, da$$

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}\rho A_0 B_0 V^2 + \int_{\text{Area}} \frac{1}{2}\rho v^2 da\right) \tag{8}$$

where the integrals are over the wire cross section. Conservation of energy and momentum imply,

$$\frac{dW}{dt} = \frac{dE_p}{dt} + \frac{dK}{dt} \qquad PA = \rho A_0 B_0 \frac{dV}{dt} \tag{9}$$

Assuming constant P, we derive[1],

$$t = \sqrt{\frac{\rho\,(B_0^2 + A_0^2)}{24\,(P/2 - 2Y/\sqrt{3})}} \int_0^{\varepsilon} \sqrt{\frac{B_0^2 e^{-2s} + A_0^2 e^{2s}}{s\,(B_0^2 + A_0^2)}}\, ds \tag{10}$$

where

$$\varepsilon = \ln\frac{A}{A_0} \tag{11}$$

The integrand may be approximated for small strains ($\varepsilon < 0.1$) by the truncated Laurent expansion in $s^{1/2}$ giving,

$$t \approx \sqrt{\frac{\rho\,(B_0^2 + A_0^2)}{3P - 4\sqrt{3}Y}} \left[\varepsilon^{\frac{1}{2}} - \frac{1}{3}\alpha\varepsilon^{\frac{3}{2}} + \frac{1}{5}\left(1 - \frac{\alpha^2}{2}\right)\varepsilon^{\frac{5}{2}} + \frac{1}{7}\left(\frac{\alpha}{3} - \frac{\alpha^3}{2}\right)\varepsilon^{\frac{7}{2}}\right] \quad \text{where } \alpha = \frac{B_0^2 - A_0^2}{B_0^2 + A_0^2} \tag{12}$$

We see from the denominator of the root that no flow occurs if $Y \geq 3^{1/2}P/4$. For an initially square wire,

$$t = \sqrt{\frac{2\rho A_0^2}{3P - 4\sqrt{3}Y}} \int_0^{\varepsilon} \sqrt{\cosh(2s^2)}\, ds$$

$$\approx \sqrt{\frac{2\rho A_0^2}{3P - 4\sqrt{3}Y}} \left(\varepsilon^{\frac{1}{2}} + \frac{1}{5}\varepsilon^{\frac{5}{2}}\right) \text{ for } A_0 = B_0 \tag{13}$$

A similar analysis is performed for a wire supported from below (lower boundary does not move) resulting in,

$$t = \sqrt{\frac{\rho\,(4B_0^2 + A_0^2)}{24P - 16\sqrt{3}Y}} \int_0^{\varepsilon} \sqrt{\frac{4B_0^2 e^{-2s} + A_0^2 e^{2s}}{s\,(4B_0^2 + A_0^2)}}\, ds$$

$$\approx \sqrt{\frac{\rho\,(4B_0^2 + A_0^2)}{6P - 4\sqrt{3}Y}} \left[\varepsilon^{\frac{1}{2}} - \frac{1}{3}\beta\varepsilon^{\frac{3}{2}} + \frac{1}{5}\left(1 - \frac{\beta^2}{2}\right)\varepsilon^{\frac{5}{2}} + \frac{1}{7}\left(\frac{\beta}{3} - \frac{\beta^3}{2}\right)\varepsilon^{\frac{7}{2}}\right] \quad \text{where } \beta = \frac{4B_0^2 - A_0^2}{4B_0^2 + A_0^2} \tag{14}$$

We see from the denominator of the root that the yield strength must be twice as high in the supported case ($Y \geq 3^{1/2}P/2$) vs. the unsupported one to avoid flow. $\beta = 3/5$ for an initially square wire.

III. COMPARISON WITH MACH2

MACH2 simulations run in planar geometry representing a solenoid wound from wire of radius $r_0 = 3.18 \times 10^{-3}$ m and having a gap between turns equal to 10% of the wire diameter are compared to our analytic free and bounded wire models. We consider wires made of half-hard Cu and 90% cold worked Glidcop™ Al_2O_3 dispersion strengthen Cu alloy AL-15[5][6] exposed to a range of magnetic field magnitudes. AL-15 has a density, compressive equation of state, conductivity, and elastic modulus similar to Cu, but a higher yield strength. The SESAME Cu equation of state table[7] and Desjarlais' modified[8] Lee-More Cu electrical conductivity model[9] are used for both materials. The Steinberg-Cochran-Guinan strain rate independent half-hard Cu model[10][11] is used for Cu's elastic-plastic behavior. The same S-C-G shear modulus is used for AL-15 but, lacking high strain rate data, AL-15's uniaxial yield strength is set to its published 90% cold worked value of 3.40×10^8 Pa. Periodic boundary conditions are invoked, corresponding to the wire being one of an infinite array of wires with center-to-center spacing of 7×10^{-3} m. The upper boundary condition is defined at $y = 0.10$ m to have a magnetic field of $\mathbf{B} = B_1\mathbf{x}$, keeping the same "up-down" convention as the analytic model. The lower boundary condition is at $y = -0.10$ cm with $\mathbf{B} = \mathbf{0}$. The simulations are for free wires only.

Figure 2 illustrates the wire outlines for three cases at the time the wires expand by 10% and, therefore, short to the neighboring turns. Figure 3 compares the time t required for the wire to spread by 10% in MACH2 simulations vs. the free and bound analytic models, as determined by Eq. 13 and Eq. 14, respectively. Here, using Eq. 2 and Eq. 11, parameters for the analytic models common to all cases considered are,

$$\begin{array}{ll} S = 7.00 \times 10^{-3}\ \text{m} & \rho = 8930\ \text{kg-m}^{-3} \\ B_0 = A_0 = 5.64 \times 10^{-3}\ \text{m} & \varepsilon = 0.0953 \end{array} \quad (15)$$

The equivalent pressure P is then determined from Eq. 2. Using the high strain rate initial yield strength from half-hard Cu's S-C-G model and the published yield strength of AL-15, we have $Y = 1.23 \times 10^8$ Pa and $Y = 3.4 \times 10^8$ Pa, respectively, for the analytic models.

We see from Fig. 3 that all three models (two analytic, one numeric) are in reasonable agreement for $B_1 \geq 50$ T, and the difference between Cu and AL-15 is small. One may infer from the small displacement of the bottom of the wire for Cu at 50 T illustrated in Fig. 3 that a bounding wall does not have time to affect the wire width

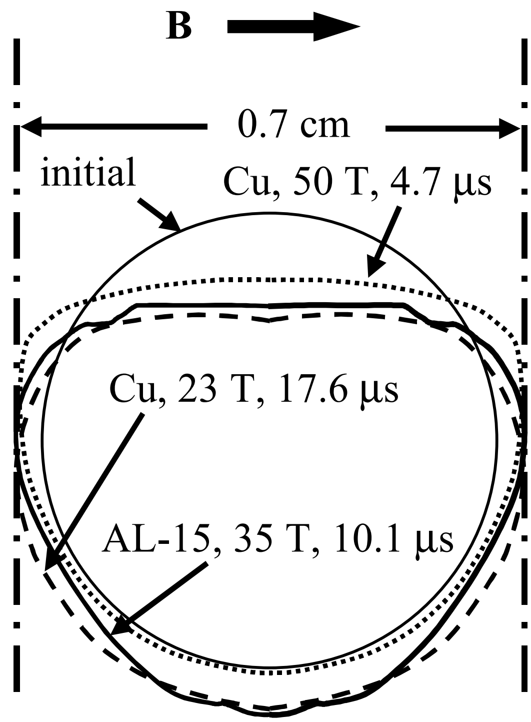

Figure 2. MACH2 results at 10% spread for two cases of solenoid wire deformation.

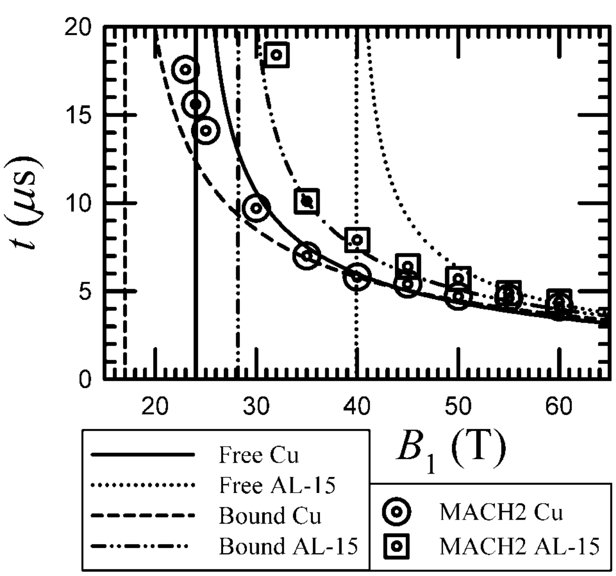

Figure 3. The time it takes a 0.318 cm radius wire in a solenoid with at 10% interwinding gap to spread 10% vs. internal magnetic field. AL-15 and Cu wires using the three theoretical models presented are compared. The vertical lines represent the analytic critical values for B_1 where no deformation occurs.

by the time the wire spreads 10% in the inertially confined (low strength) limit. Both analytic models reflect this accurately; they differ by only about 10% for $P >> Y$ for slightly strained square wires. It comes as a surprise for lower magnetic fields, however, that MACH2 agrees much better with the analytic bounded wire model than with the free wire model that was originally intended to represent it. It appears that when strength does play a significant role in impeding expansion, the bound wire model provides the best correlation. Figure 2 suggests an explanation. The wire's shape at 10% expansion varies little with the relative values of Y and B_1. Furthermore, the deformation occurs primarily in the top half before the bottom is substantially displaced. Placing a lower boundary on the analytic model apparently offsets the error introduced by the unrealistically homogeneous strain assumed.

Such deformation localization decreases, and the discrepancy between the bound model and MACH2 increases, as the magnetic field is lowered toward the critical value where no deformation occurs. The Cu wire exposed to 23 T, for example, is more elliptical than the other cases at 10% expansion, as evidenced by the greater bulge in the lower half of the wire shown in Fig. 3. This implies a more homogeneous strain profile. With the plastic work, therefore, more uniformly distributed, the wire more efficiently resists deformation than in cases where the magnetic field is well above its critical value.

IV. CONCLUSIONS

Real solenoid currents can rarely be approximated as rising abruptly to a constant value due to inductance. The analytic free and bound wire models may be generalized to account for a time dependent current and, therefore, P by numerical integration of Eqs. 9. Since the bound wire model provides a good correlation to MACH2 simulations for a broad range of constant pressures, it is reasonable to conclude that it takes into account inertial and material strength effects with reasonable accuracy. The correlation, therefore, is likely to be good for time dependent pressures too. This should be especially true for cases where the peak effective P is significantly higher than the bound model's critical value of $2Y/3^{1/2}$.

The good correlation between the bound wire model and MACH2 depends on the fact that for a round wire, where magnetic pressure gradients are directed toward the center of the wire, the deformation is distributed broadly over a significant portion of the cross section. If an actual rectangular wire is use, however, this is unlikely to be the case. A similar magnetohydrodynamic simulation with material strength treatment performed for a single turn solenoid with its winding having a rectangular cross section results in the inner surface of the solenoid deforming locally into a mushroom shaped profile[12], much as in a peened rivet, when it is used to create a high magnetic field. It is with some irony, then, that the rectangular models presented can be expected to be inaccurate for actual wires of rectangular cross section.

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