Testable restrictions of Pareto optimal public good provision

Susan K. Snyder[*]
Department of Economics
Virginia Polytechnic Institute & State University
Blacksburg, VA 24061-0316
sksnyder@vt.edu

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Abstract

This paper is a theoretical examination of the testable restrictions of Pareto optimality in an economy with public goods. Testable restrictions of a model of public sector Pareto optimality and private sector competitive equilibrium are developed through the algebraic method of quantifier elimination. In contrast to other methods for deriving tests for the Pareto optimality of allocations with public goods, quantifier elimination does not require ad hoc assumptions about individual public good demand. The only assumptions needed are that individuals have continuous, strictly monotonic, strictly quasi-concave utility functions defined over private and public goods. This method also requires relatively less observation of individual-level data; the testable restrictions derived in this paper are defined over a finite series of aggregate-level data, production data, and individual after-tax income.

Despite the relatively sparse data requirements and the minimal assumptions about preferences and technology, these restrictions are not vacuous. Thus they provide a strong theoretical basis for empirical work that tests the hypothesis of Pareto optimality in economies with public goods in a broad variety of situations.

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This paper also discusses the usefulness of these results in the context of aggregate demand. A subset of the general equilibrium model restrictions can be used to test whether aggregate demand of public and private goods is optimal given the preferences of its members. It is proven that the hypothesis of Pareto optimality in this context can be meaningfully tested using group-level data and individual level after-tax income, with no parametric assumptions about individual public good demand.

Keywords: public goods, Pareto optimality, quantifier elimination, Lindahl equilibrium, testable restrictions, revealed preference.

JEL: D51, H41

1 Introduction

Economic theory predicts that, in general, public goods will be provided at a suboptimal level by private mechanisms, whereas informational problems will tend to lead to inefficiencies when public goods are provided by the public sector. Thus models of efficient public good provision are often seen more as normative prescriptions rather than positive descriptions of how public goods are allocated (Laffont, 1988). Whether a particular allocation is efficient, however, is ultimately an empirical question.

This paper focuses on the empirical implications of Pareto optimal behavior in an economy with public goods. It demonstrates a method to test whether public good provision is consistent with Pareto optimality without having individual-specific information about public good preferences. The only assumptions required are that agents have preferences representable by continuous, strictly monotonic, strictly quasi-concave utility functions defined over private and public good consumption. Additionally, the only individual-specific information required is individual after-tax income (or income net of public good contributions).

This paper derives testable restrictions of Pareto optimal public good provision for an economy with constant returns to scale technology and two traders for two observations of aggregate production of private and public goods, prices, aggregate endowments, aggregate taxes, and individual after-tax incomes. These testable restrictions are nonparametric, nonstochastic conditions defined over discrete observations of data. In form, they are analogous to revealed preference tests for individual demand data such as the Weak Axiom of Revealed Preference. Implementing the tests is simply a matter of checking whether data satisfy a series of polynomial inequalities.

The testable restrictions are derived by adapting the techniques demonstrated in Brown and Matzkin's work (1996) on the empirical applications of equilibrium models. They prove that non-vacuous testable restrictions of competitive equilibrium exist on the equilibrium manifold; that is, one can test equilibrium behavior on a series of observations of prices and individual endowments from an economy. A key element in their proof is the application of the algebraic method of quantifier elimination. This method allows the systematic derivation of necessary and sufficient conditions for a given set of observable variables to be consistent with the model, even when there exist important unobservable variables in the model's equilibrium conditions. In the case of an economy with public goods, Lindahl prices are inherently unobservable, but through quantifier elimination we can derive the necessary and sufficient conditions of the model without observing them.

If these restrictions are satisfied, we say the data is *rationalized*: there exists an economy such that this data is consistent with Pareto optimal public good provision in some economy. If the restrictions are not satisfied, there does not exist such an economy; the data is not consistent with Pareto optimal public good provision. Thus these are necessary and sufficient conditions; they describe all the empirical implications of the model given our assumptions about what data is observable.[1]

The relatively sparse data requirements of these restrictions means that they will be of use in a broad variety of contexts. How will we interpret the results of these tests when applied to data? The relatively few assumptions that go into creating these testable restrictions make a negative result more robust than results that depend on a particular parametric specification of preferences or technologies. Thus if we cannot rationalize the data, this would provide some evidence supporting the numerous theories that predict the free-rider problem or informational problems will lead to inefficiencies in public good provision.

If the data is rationalized with these tests, a number of interpretations are possible. We could view this as evidence that the free-rider problem or informational problem has been overcome by the particular group at hand. We could further interpret this as support for some sort of cooperative behavior.

While these restrictions are for a general equilibrium model with production, one can use a subset of the restrictions to test the Pareto optimality of aggregate demand. This is likely to be more appropriate for many empirical situations. We need only two observations of the aggregate consumption and prices of private goods and public goods, aggregate taxes or contributions, aggregate before-tax income, and individual private good expenditures, to test for Pareto optimal provision of public goods in conjunction with optimal individual private good choices.

These tests should provide a useful counterpart to existing empirical work on testing Pareto efficiency of public good provision. Empirical tests of the efficiency of public good provision have generally proceeded with additional assumptions about the form of individual public good demand. Sandler and Murdoch (1990) apply a formal test of efficiency of private provision of public goods by examining defense

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[1] If there are more than 2 observations these restrictions can still be used to refute Pareto optimality; if any pair of observations fail to satisfy the restrictions the data is not consistent with Pareto optimal public good provision. They are no longer sufficient conditions, however.

spending by NATO members. Khanna (1993) applies similar methods to examine the supply of agricultural research by American states.[2] Using econometric specification of log-linear demand for public goods, both papers find little support for efficient behavior.

The next section presents the public competitive equilibrium model that provides the framework for the derivation of tests of efficient public good provision. The testable restrictions of this model are derived in section 3. Section 4 discusses some general implementation issues and applies the restrictions of Pareto optimal public good provision to derive tests of efficient household behavior.

2 Public competitive equilibrium

In a Lindahl equilibrium, each individual must pay taxes based on their marginal benefit of the public good. There may be other tax schemes, however, that are compatible with Pareto optimality in the presence of public goods — and because redistribution is so often a conflating issue in tax schemes, we can study a much broader range of situations by allowing that redistribution may exist. A less restrictive alternative to the Lindahl equilibrium is the politico-economic equilibrium, first suggested by Wicksell (1896) and formalized by Foley (1967) as the public competitive equilibrium. In this model, consumers behave competitively with respect to private goods. Public goods, however, are provided by the public sector. The public sector chooses public goods and lump-sum taxes to finance them such that there is no other public sector proposal that would be preferred by every individual.[3]

Foley (1967) proves that public competitive equilibria are Pareto optimal, and there exist prices and taxes such that, given endowments, a Pareto optimal allocation is a public competitive equilibrium (see also Milleron (1972)). Testing whether data from an economy is consistent with public competitive equilibrium will provide a way of testing whether the economy has reached a Pareto optimal allocation of public goods.

2.1 The Model

Consider an economy with T consumers, K pure public goods and J private goods.[4] Each consumer t has preferences over public and private goods represented by a

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[2] See also McGuire and Groth (1985) for a theoretical basis for the tests used in both these papers.

[3] While agents can choose different tax schemes within the model with a unanimous vote, there is no specification as to how the status quo tax scheme is chosen. Thus rather than being a true equilibrium model, the public competitive equilibrium is more a description of public sector Pareto optimality combined with private sector competitive equilibrium (see Milleron (1972), also Hammond (1992)).

[4] This description of the model is adapted from Foley (1970).

continuous, strictly monotonic, strictly quasi-concave utility function $U_t(x_t; y_t)$, with $x_t \in R^K_+$, $y_t \in R^J_+$. Consumption of the public goods is equal across all consumers (there is no free disposal): $x_t = x$ for $t = 1, ..., T$, while aggregate consumption of private goods is equal to the sum of private consumption: $y = \sum_{t=1}^{T} y_t$.

Consumers have endowments of private goods $\omega_t \in R^J_+$, for $t = 1, ..., T$, with aggregate endowment represented by ω. We assume these endowments can be used to produce public goods and private goods, with net output of private goods represented by $z = y - \omega$, and output of public goods represented by x. We assume public goods are not used in production.

Production technology is assumed to be constant returns to scale.[5] Formally, the set of all technically possible production plans is Z, and we assume:

A1. Z is a closed, convex cone.

A2. If $0 \neq (x; z) \in Z$ then $(x; z) \ngeq 0$.

Definition 1 *A Feasible Allocation is a set of vectors* $(x; \{y_t\}_{t=1}^{T})$ *such that* $(x; y - \omega) \in Z$.

There exists a government that can purchase public goods for the use of the economy's members and also has the power to tax members to pay for the public goods and to redistribute income (taxes can be negative). Each consumer makes a tax payment (or receives a subsidy) of τ_t.

It is straightforward to show that the equilibrium conditions of the public competitive equilibrium model are the same as a Lindahl equilibrium with transfers. Thus if the economy is at a Pareto optimal allocation, there exist Lindahl prices such that consumers act *as if* they were choosing public and private goods subject to prices and a full income budget constraint. Government choice is then reduced to a balanced-budget condition.

In the following definition of a public competitive equilibrium the prices q_t correspond to the Lindahl prices. We also distinguish between monetary transfers (τ_t) and full income transfers ($S_t = \tau_t - q_t x_t$).

Definition 2 *A Public Competitive Equilibrium is a feasible allocation* $(x; \{y_t\}_{t=1}^{T})$, *prices* $(q; p) \geq 0$, *and taxes* $\{\tau_t\}_{t=1}^{T}$ *such that consumers, producers and the government act as if:*

2.1 Each consumer solves the maximization problem:

$$\max_{(x_t; y_t)} \quad U_t(x_t; y_t) \quad s.t. \quad q_t x_t + p y_t \leq p\omega_t + S_t$$

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[5] See Milleron (1972) for a version of Foley's model with less restrictive assumptions on the production sector: public goods can be used in production and can be part of the endowments, and technology is not restricted to be CRS.

2.2 Producers solve the maximization problem:

$$\max_{(x;z)}(q;p)\cdot(x;z) \quad s.t. \quad (x;z)\in Z$$

2.3 The government chooses $\{\tau_t\}_{t=1}^{T}$ *such that:*

a. $q\,x=\sum_{t=1}^{T}\tau_t$

b. $\tau_t=q_t x-S_t \quad t=1,...,T$

2.4 Public good restrictions are satisfied:

a. $x_t=x \quad t=1,...,T$

b. $\sum_{t=1}^{T}q_t=q$

3 Testable restrictions

Testable restrictions of equilibrium models are most often developed through the method of infinitesimal comparative statics. For example, the endogenous variables of a model are written as functions of the exogenous variables; the model implies certain properties for these functions, and we can then test whether data is consistent with the model by testing whether the data is consistent with these properties. It is well-known that the standard model of competitive equilibrium possesses no interesting comparative statics properties of this sort (see, for example, Kehoe (1987)).

In contrast, Brown and Matzkin (1996) show that there exist nonparametric testable restrictions of general equilibrium models defined over discrete observations of data. They show these testable restrictions exist through extending the results of revealed preference theory, developed for the theory of individual and firm-level optimization, to general equilibrium models. Their method makes use of the discrete nature of observable data and the algebraic techniques that can be applied as a result.

Rather than making assumptions about the particular form of individual preferences in the economy, Brown-Matzkin generate all the restrictions on aggregate behavior in equilibrium that result from the non-negativity of the unobserved individual demand. Using the duality of public good and private good models, we can generate restrictions of aggregate public good provision in equilibrium that result from the assumption of non-negativity of the unobservable personalized public good prices. This is a substantive assumption. We have assumed there is no free disposal of public goods; therefore limiting personalized prices to be non-negative implies that in equilibrium individual public good marginal benefit from each public good must be non- negative. That is, at the margin, consumers always get positive benefit from every public good — there are no "public bads" from any consumer's standpoint. Without the non-negativity assumption, however, there will generally be no interesting restrictions on aggregate public good demand (see Chiappori (1990)).[6]

This paper will follow the method used in Brown-Matzkin to derive nonparametric testable restrictions of the public competitive equilibrium model. Their method has two parts: define the equilibrium conditions of the model as a set of polynomial inequalities; then apply the algebraic technique of quantifier elimination to rewrite the equilibrium conditions to involve only variables that we can observe. These resulting conditions will be the testable restrictions of the model.

The public competitive equilibrium model has essentially three components: individual utility maximization conditions; profit maximization conditions; and summing-up conditions. We can re-write the equilibrium conditions of the model as a series of polynomial inequalities in the model's variables, without making further assumptions about utility or production functions, by applying Afriat's Theorem and its variants. Afriat's Theorem states that there exists a finite set of polynomial inequalities ("Afriat inequalities") defined over discrete observations of individual consumption data and other unobservable variables (such as utility levels) that provide necessary and sufficient conditions for the data to have been generated by the maximization of a non-satiated utility function (Afriat (1967); also Varian (1982)).

Thus in describing a public competitive equilibrium's empirical implications, we can replace the utility maximization condition with an equivalent set of polynomial inequalities, the Afriat inequalities. Analogous results for profit maximization allow us to replace profit maximization condition with equivalent polynomial inequalities as well. Doing so will depend on the assumption that we can observe a series of allocations of the economy in question where individual preferences and production technologies do not change over time.

The significance of rewriting the equilibrium conditions as polynomials is that we can then apply the technique of quantifier elimination to show that testable restrictions exist given the data we can observe. Given a finite set of polynomial inequalities in observable and unobservable variables, the Tarski-Seidenberg theorem proves that there exists an equivalent finite set of polynomial inequalities that does not involve the unobservable variables, and also provides a finite algorithm for computing the equivalent system through quantifier elimination (Tarski, 1951). Quantifier elimination is the process of eliminated "quantified" variables — in our case, the unobservable variables are quantified in that they appear in conjunction with the existential quantifier ("there exists"). Quantifier elimination can be used to show that unobservable variables (such as Lindahl prices) can be "eliminated" from the equilibrium conditions of the model, leaving behind non-vacuous, nonparametric testable restrictions of the public competitive equilibrium model involving data we can potentially observe.

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[6] Alternatively, one could derive non-trivial restrictions by assuming free disposal of public goods, which implies that the individual marginal benefit of any public good will be non-negative.

Rather than applying the Afriat inequalities for our model, we will use revealed preference tests instead. It has been proven that the Afriat inequalities are equivalent to the Generalized Axiom of Revealed Preference (GARP), a generalization of Samuelson's Weak Axiom of Revealed Preference (WARP) (see Varian (1982)).[7] Thus one can use the Afriat inequalities or GARP to create testable restrictions of a model with utility maximization; the advantage of GARP for our purposes is that it leads to linear quantifier elimination problems, while the Afriat inequalities lead to bi-linear problems (Snyder, 1995). For our tests we will make use of a theorem due to Matzkin and Richter (1991) that shows there is a slight modification of the Afriat inequalities that is equivalent to Houthakker's Strong Axiom of Revealed Preference (SARP), the transitive version of Samuelson's Weak Axiom. The difference between SARP and GARP is essentially SARP restricts demand to be single-valued.

On the production side, the Weak Axiom of Profit Maximization (WAPM) provides necessary and sufficient conditions for profit maximizing behavior in the form of polynomial inequalities defined over a discrete series of data.[8] We assume technology is constant returns to scale; therefore profits must be zero. Thus necessary and sufficient conditions for there to exist a production set defined by constant returns to scale technology such that producers maximize profits subject to feasibility constraints defined by this production set combine the WAPM with the zero profit condition (Hanoch and Rothschild (1972), see also Varian (1984), Theorem 6).

If we had a data set that contained all these variables, then we could use these conditions to test whether the data is consistent with public competitive equilibrium. It is not possible to observe all these variables, however, particularly the Lindahl prices, q_t. Additionally, it may be difficult to observe highly disaggregate data, such as individual consumption of private goods.

Theorem 1 presents the public competitive equilibrium model in polynomial form. Let after-tax income for consumer t in period r be defined as $I_t^r = p^r \omega_t^r - \tau_t^r$, and, in general, let the notation $\{I_t^r\}$ represent $\{I_t^r\}_{t=1}^T$.

Theorem 1 *Let the collection* $\langle x^r, y^r, q^r, p^r, \omega^r, \{I_t^r\}, \tau^r \rangle$ *of non-negative vectors of variables be given for* $r = 1, ..., R$. *Then there exist continuous, strictly monotonic, strictly concave utility functions* $\{U_t\}$ *and a closed convex conical, negative monotonic production set* Z *such that this data is consistent with a series of Public Competitive Equilibria for the economy* $(\{U_t\}, Z, \{\{\omega_t^r\}_{t=1}^T\}_{r=1}^R)$, *if and only if there exist* $\langle \{y_t^r\} \geq 0, \{q_t^r\} \geq 0 \rangle$ *such that:*

1. $\langle x^r, y_t^r, q_t^r, p^r, I_t^r \rangle$ *satisfy the Strong Axiom of Revealed Preference and the individual budget constraint,* $p_t^r y_t^r = I_t^r$, *for each consumer* t.

2. $\langle x^r, y^r, q^r, p^r, \omega^r \rangle$ *satisfy profit maximization and technology restrictions:*

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[7] Afriat derived equivalent conditions he called "cyclical consistency".

[8] For origins of WAPM see Varian (1984).

   a. $0 = (q^r; p^r) \cdot (x^r; y^r - \omega^r) \geq (q^r; p^r) \cdot (x^s; y^s - \omega^s)$ *for* $r, s = 1, ..., R$, $r \neq s$.

   b. *If* $y^r - \omega^r \neq 0$ *then* $y^r - \omega^r \not\geq 0$ *for* $r = 1, ..., R$.

3. $\langle x^r, q^r, \tau^r \rangle$ *satisfy the public good budget constraint:*

$$q^r x^r = \tau^r \quad r = 1, ..., R.$$

4. $\langle y^r, q^r, p^r, \{y_t^r\}, \omega^r, \{I_t^r\}, \{q_t^r\}, \tau^r \rangle$ *satisfy:*

   a. $\sum_{t=1}^{T} y_t^r = y^r \quad r = 1, ...R.$

   b. $\sum_{t=1}^{T} q_t^r = q^r \quad r = 1, ...R.$

   c. $\sum_{t=1}^{T} I_t^r = p^r \omega^r - \tau^r \quad r = 1, ..., R.$

Proof: Follows directly from Afriat's Theorem, Matzkin and Richter (1991), and Varian (1984), Theorem 6.

Comment 1: Note that if the conditions of the theorem are satisfied, there exist strictly concave utility functions consistent with a public competitive equilibrium, though we only assumed strict quasiconcavity up to this point. This derives from Afriat's theorem: if a finite series of individual consumption data is consistent with maximization of a non-satiated utility function, it is also consistent with maximization of a concave, continuous, monotone utility function. Violations of convexity cannot be detected with finite data. Similarly, assuming strict concavity in this context is no more restrictive than assuming strict quasiconcavity. For similar reasons, the addition of negative monotonicity of the production set is no more restrictive than our previous assumptions, given finite data sets.

Comment 2: The strong axiom of revealed preference can be written as a finite set of disjunctions of polynomial inequalities. All the other conditions of the theorem are also finite sets of polynomial inequalities or equations.

Theorem 1 provides a statement of public competitive equilibrium behavior in the form of a finite set of polynomial inequalities defined over a finite set of variables. Moreover, we have explicitly assumed that $\langle \{y_t^r\}, \{q_t^r\} \rangle$, the Lindahl prices and individual private good consumptions, are unobserved, while $\langle x^r, y^r, q^r, p^r, \omega^r, \{I_t^r\}, \tau^r \rangle$, the public good consumptions, aggregate private good consumptions, market prices, aggregate endowments, individual after-tax incomes and aggregate tax payments, are observed. Thus theorem 1 presents the equilibrium conditions of public competitive equilibrium behavior as a finite set of polynomial inequalities in observed and unobserved variables. What remains is to eliminate the unobservable variables from the system.

There are three possible outcomes when quantifier elimination is applied to a system such as the one in theorem 1. One possibility is the equivalent system reduces to $1 \equiv 0$, meaning it is impossible to observe data consistent with the model; the theory is empirically inconsistent.[9] Another possibility is the system reduces to $1 \equiv 1$, meaning it is impossible to observe data not consistent with the model; the theory imposes no testable restrictions on the set of observable variables. The third possibility is the system reduces to a finite set of polynomial inequalities involving the observable variables. It is possible to observe data that satisfies these inequalities, and it is possible to observe data that does not satisfy these inequalities. In this case we say testable restrictions of the model exist, given the set of observable variables (Brown and Matzkin, 1996).

While Tarski-Seidenberg provides a finite algorithm for quantifier elimination, it is doubly exponential and so is not practical. Fourier-Motzkin elimination is a widely-known algorithm that is applicable to linear systems such as the ones in this problem; however, it is also doubly exponential in application.[10] Lacking any practical quantifier elimination algorithms to apply to this problem, we will focus on very simple models of public competitive equilibrium. We will derive the testable restrictions of this model for two observations of an economy with two agents, a and b, and any finite number of public and private goods.[11] For the case of one public good and one private good, we will also show that these restrictions are non-vacuous.

In the following lemma we consider the consumption side of this problem:

Lemma 1 *Let the collection* $\langle x^r, y^r, q^r, p^r, I_a^r, I_b^r \rangle$ *of non-negative variables be given for* $r = 1, 2$*. Then there exist continuous, strictly monotonic, strictly quasi-concave utility functions* U_t *such that this data is consistent with each consumer satisfying the Weak Axiom of Revealed Preference (WARP) and their individual budget constraint, Lindahl prices summing to the market price, and individual private good consumptions summing to aggregate private good consumption if and only if:*

For some $r, s = 1, 2$, $r \neq s$*, either:*

$$\textbf{I} \qquad [E_a^{rs} > I_a^r \ \textit{AND}\ E_b^{rs} > I_b^r \ \textit{AND}\ (q^r; p^r) \cdot (x^s - x^r; y^s - y^r) > 0]$$

or:

$$\textbf{II} \qquad [E_a^{rs} > I_a^r \ \textit{AND}\ E_b^{sr} > I_b^s]$$

Where $E_t^{rs} = \max_{\nu,\mu} \mu \cdot (x^s - x^r) + p^r \nu \quad s.t. \quad p^s \nu = I_t^s, \ 0 \leq \nu \leq y^s, \ 0 \leq \mu \leq q^r.$

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[9] Empirical consistency is distinct from the traditional use of "inconsistency" in general equilibrium theory, see Snyder (1995).

[10] Fourier-Motzkin is a technique similar to Gaussian elimination. For some explication of the method and its uses, see Dantzig and Eaves (1973).

[11] The restrictions could be used with more than two consumers, but the usual problems of representative agent models are introduced. See Kirman (1992).

Proof: See Appendix.
Comment: WARP is equivalent to SARP for two observations.

One could think of these conditions as analogous to WARP. WARP tests individual behavior for utility maximization; these conditions test a combination of aggregate behavior (public good demand, private good demand) and individual behavior (after-tax income) for individual utility maximization.

It is well-known that individual utility maximization does not imply collective behavior that resembles utility maximization, and that can be seen quickly from these restrictions. Note that if efficiency required collective behavior to resemble that of an individual utility maximizer, then aggregate demand would satisfy WARP. The last restriction in Condition I implies that aggregate consumption in period r is not revealed preferred to aggregate consumption in period s — if this condition is satisfied, then aggregate demand satisfies WARP. For aggregate demand to satisfy WARP is not a sufficient condition for efficiency, however; the other restrictions in Condition I must also be satisfied. Moreover, for aggregate demand to satisfy WARP is not a necessary condition; if the data satisfies Condition II aggregate demand does not have to satisfy WARP. Thus restrictions individual demand to satisfy WARP does not imply aggregate demand satisfies WARP.

Interpretation of these restrictions in terms of the usual demand restrictions we are familiar with are difficult because these restrictions depend not on aggregate demand alone but on the (observed) income distribution. These restrictions simply define bounds such that it is possible for each individual to be satisfying the axiom of revealed preference. The non-vacuousness of the restrictions comes from the non-negativity restrictions on individual consumption and Lindahl prices. Given this level of observability, it is always possible for each individual to be maximizing utility given any data; it is not always possible, however, for both individuals to be maximizing utility in equilibrium (that is, there may be individual private valuations of the public good such that utility maximization is satisfied, but the individual private valuations of the public good will not sum up to the public good price).

Figure 1 provides an example of data that does not satisfy the restrictions. We observe public good consumption; by observing prices and each consumer's income we can derive private good consumption for each consumer. This picture represents a situation where both consumers get equal amounts of the private good, and by definition both consume the same amount of the public good. The picture represents consumer a's situation: a's period 1 consumption is plotted at (x^1, y_a^1) and at (x^2, y_a^2) in period 2. The picture would look exactly the same if we instead plotted consumer b's consumption. We observe the price of the private good but we do not observe the Lindahl price of the public good for each consumer. Thus we are free to choose the slopes of the price lines for each consumer, with the restriction that the Lindahl prices must be non-negative and must sum to the observed market public good price (normalized to 1). Then the data represented by this picture is consistent with the public good restrictions only if we can draw feasible price lines such that each consumer satisfies WARP.

Let $r^1 = q_a^1/p^1$ be the slope of budget line 1, and $r^2 = q_a^2/p^2$ be the slope of budget line 2. Then $0 \leq r^1 \leq 1/p^1$ and $0 \leq r^2 \leq 1/p^2$. The limit-cases are drawn in figure 1. Note that for all feasible public good prices, consumer a's period 2 consumption is revealed preferred to her period 1 consumption. Thus the only way this consumer can satisfy WARP is if we choose q_a^1 such that her period 1 consumption is not revealed preferred to her period 2 consumption. This is possible; for example, a Lindahl price of 0, or close to 0, would satisfy this requirement. In general, it is always possible to choose Lindahl prices such that WARP is satisfied for one consumer.

Suppose we choose a low enough Lindahl price for consumer a in period 2 so that her period 2 consumption bundle will no longer be revealed preferred to her period 1 consumption bundle. With the data illustrated here, a Lindahl price of 1/2 is too high – at this price, $a's$ period 1 consumption is revealed preferred to her period 2 consumption. Thus q_a^1 must be less than 1/2. But because the Lindahl prices must sum to 1, this means choosing a Lindahl price for consumer b that is higher than 1/2. However, consumer b faces the same situation as consumer a – b must have a Lindahl price less than 1/2 in order to satisfy WARP. Thus while there exist Lindahl prices such that either consumer can satisfy WARP, there do not exist Lindahl prices such that each consumer can satisfy WARP with Lindahl prices that sum to the market price for the public good. The data represented by this picture is not consistent with Pareto optimal public good choice.[12]

The following theorem uses lemma 1 to derive testable restrictions of the public competitive equilibrium model.

Theorem 2 *Let the collection* $\langle x^r,\, y^r,\, q^r,\, p^r,\, \omega^r,\, I_a^r,\, I_b^r,\, \tau^r\rangle$ *of non-negative vectors of variables be given for* $r = 1, 2$. *Let:*

$$D_1 = \langle x^r,\, y^r,\, q^r,\, p^r,\, I_a^r,\, I_b^r\rangle$$

$$D_2 = \langle x^r,\, y^r,\, q^r,\, p^r,\, \omega^r\rangle$$

$$D_3 = \langle x^r,\, q^r,\, p^r,\, \omega^r,\, I_a^r,\, I_b^r,\, \tau^r\rangle$$

Then there exist continuous, strictly monotonic, strictly concave utility functions $\{U_t\}$ *and a closed convex conical, negative monotonic production set* Z *such that this data is consistent with a series of Public Competitive Equilibria for the economy* $(\{U_t\}, Z, \{\{\omega_t^r\}_{t=1}^T\}_{r=1}^R)$, *if and only if:*

1. *D_1 satisfy the conditions of Lemma 1.*

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[12] See appendix for a numerical example of data that is not consistent with Pareto optimality.

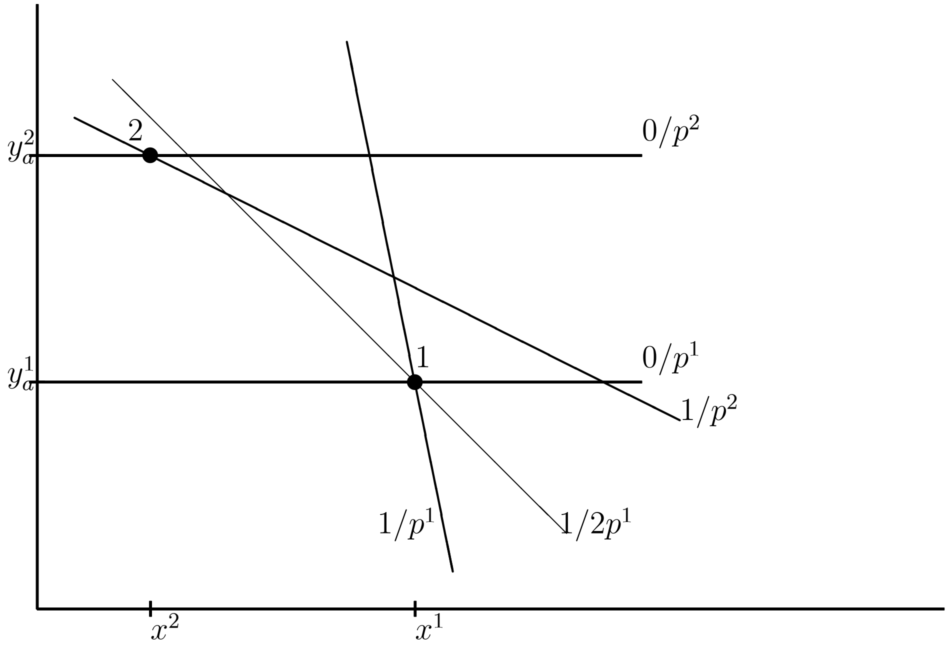

Figure 1: Data that does not satisfy the restrictions

2. *D_2 satisfy profit maximization and technology restrictions:*

   a. $0 = (q^r; p^r) \cdot (x^r; y^r - \omega^r) \geq (q^r; p^r) \cdot (x^s; y^s - \omega^s)$ *for* $r, s = 1, 2$*,* $r \neq s$*.*

   b. *If* $y^r - \omega^r \neq 0$ *then* $y^r - \omega^r \ngeq 0$ *for* $r = 1, 2$*.*

3. *D_3 satisfies the following constraints for $r = 1, 2$:*

   a. $\tau^r = q^r x^r$

   b. $I_a^r + I_b^r = p^r \omega^r - \tau^r$

Proof: Condition (3) is straightforward, as it involves only observable variables. Condition (2) follows from Theorem 1, and condition (1) follows directly from Lemma 1.
Comment: Since the Lemma 1 conditions are non-vacuous for the case of one public good and one private good, the testable restrictions of the public competitive equilibrium are non-vacuous for that case also.

Theorem 2 provides the testable restrictions of public competitive equilibrium, given our assumptions about the observability of the model's variables. Thus they form testable restrictions of Pareto optimality in a general equilibrium model with public goods.

There may be situations where it is clearly not appropriate to test for efficiency in a general equilibrium model, but it may be appropriate to test for efficiency of aggregate demand. Suppose we had a group of consumers who faced exogenous public and private good prices, and who were taxed (or made contributions) for the collective purchase of public goods. We could test the Pareto optimality of the group's choice of public and private goods using a subset of the restrictions of the model of public competitive equilibrium — we simply exclude the production sector restrictions on technology and profit maximization. In this nonparametric, revealed preference based method of deriving testable restrictions, it is irrelevant that prices are endogenous in the public competitive equilibrium model and exogenous in the aggregate demand model.

Corollary 1 *Let the collection* $\langle x^r, y^r, q^r, p^r, \omega^r, I_a^r, I_b^r, \tau^r \rangle$ *of non-negative vectors of variables be given for* $r = 1, 2$*. Let:*

$$P_1 = \langle x^r, y^r, q^r, p^r, I_a^r, I_b^r \rangle$$

$$P_2 = \langle x^r, q^r, p^r, \omega^r, I_a^r, I_b^r, \tau^r \rangle$$

Then there exist continuous, strictly monotonic, strictly concave utility functions $\{U_t\}$ *such that this data is consistent with Pareto optimal demand for the economy* $(\{U_t\}, \{\{\omega_t^r\}_{t=1}^T\}_{r=1}^R)$*, if and only if:*

1. *P_1 satisfy the conditions of Lemma 1.*

2. *P_2 satisfies the following constraints for $r = 1, 2$:*
   - *a.* $\tau^r = q^r x^r$
   - *b.* $I_a^r + I_b^r = p^r \omega^r - \tau^r$

4 Implementation and Application

4.1 Issues in Implementation

One advantage of these tests is that they require little data to implement. To test efficiency of demand, we require two observations over time of an economy's allocations or a group's consumption choices, prices, and the individual after-tax incomes. For example, we could use these conditions to test whether police protection in a town is Pareto optimal with two observations (perhaps of annual data) of the number of police officers, the town's aggregate consumption of private goods, the "price" of a police officer (perhaps measured by salary or inferred from expenditures on police protection and number of officers used), aggregate tax collection, aggregate income, and expenditure on private goods for two types of citizens of the town. Or, we could test whether an interest group purchases a Pareto optimal amount of advertising for a candidate it wishes to elect. To test this proposition we might use data consisting of two observations (perhaps monthly) of interest group expenditures on advertising, the quantity of advertising produced (perhaps measured by an estimate of the number of people who viewed the advertising), the group members' aggregate consumption of private consumption goods, aggregate contributions, aggregate income, and expenditures on private goods for two types of interest group members.

The assumption that preferences and technology don't change over time is crucial for these tests; additionally, we assume that consumption and production choices are independent over time. Alternatively, one could apply the restrictions to a cross-sectional analysis, where it would be necessary to assume that members of two groups or economies have the same preferences, while having different endowments or facing difference prices.

The tests as formulated in this paper are non-stochastic. While this makes the tests easy to apply, it also creates problems in the interpretation of situations where data does not satisfy the restrictions. If the restrictions are not satisfied, it is not clear whether we should interpret this as a rejection of the model, or as evidence for some measurement error, or as evidence that there is some stochastic element in behavior unaccounted for. These tests may be particularly useful as specification tests in preparation for doing further econometric work. Note however that the method outlined here for deriving testable restrictions is capable of accommodating stochastic elements, see Brown and Matzkin (1994) for work in this direction.

Strictly speaking, the tests derived in this paper are applicable only to models with two consumers. One could assume a model with two representative consumers, each representing a distinct group within the economy, though the lack of microfoundations for the representative consumer model may make this an unappealing assumption.[13]

4.2 Intra-household decision making

One possible application of this work is in the area of household decision making. Traditionally households have been treated as a single utility-maximizing agent (the "neoclassical" model of household behavior). Recent years have seen the development of numerous models of household decisions that are based on individual behavior within the household, modeling outcomes as the result of either cooperative or non-cooperative bargaining between household members (see, for example, McElroy and Horney (1981), Manser and Brown (1980), Lundberg and Pollak (1994)). Testing whether these models adequately explain household behavior can be difficult, however, as we generally observe market decisions at the household level rather than at the individual level — we generally do not observe the division of goods within the household.

Chiappori (1988) proposed an extremely general model of cooperative bargaining within the household (the "collective rationality" model), stipulating only that outcomes be Pareto optimal. Individuals within the household get utility from their own consumption and leisure, but their consumption and leisure may also generate positive externalities for the other person in the household. One could think of the collective rationality model as a simple model of Pareto optimality with public goods, as each individual's consumption and leisure become public goods within the household.

Chiappori (1988) shows that there are non-vacuous testable restrictions of the collective rationality model on data that can feasibly be observed: household-level consumption, individual labor supplies, prices and wages. Chiappori's tests are in the form of finding whether a set of bilinear inequalities has a solution — if the program has a solution, then the data satisfy the model, if not, the data do not satisfy the model.

By applying Corollary 1 to this problem, we can derive equivalent testable restrictions as a set of polynomial inequalities defined only over the observable variables. In other words, Chiappori's program will have a solution if and only if these polynomial inequalities are satisfied.

To illustrate, consider the following variation of Chiappori's model. A household

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[13] The use of a representative consumer is consistent with individual utility maximization only with highly restrictive assumptions; for example, to avoid making further restrictions (such as homotheticity) on the form of individual preferences, one would have to assume identical preferences and identical incomes within groups (see Kirman and Koch (1986)).

is composed of two members, a and b. Let C^r denote the household-level quantity of some consumption good in period r, which has a price normalized to one. Let L_a^r and L_b^r denote, respectively, the leisure of the members of the household, and w_a^r and w_b^r their wages. Assume labor supply is observable for each member of the household, and all time not spent supplying labor to the market is leisure time. The household also receives exogenous non-labor income Y^r.

Assume that individual consumption generates externalities within the household; that is, each individual's consumption, c_a and c_b, is a public good within the household. Each individual has preferences representable by strictly monotone, strictly quasi-concave utility functions: $U_t(c_a, c_b, L_t)$.

Corollary 2 *Let the collection* $\langle C^r, Y^r, L_a^r, L_b^r, w_a^r, w_b^r \rangle$ *of non-negative variables be given for* $r = 1, 2$. *Then there exist continuous, strictly monotonic, strictly quasi-concave utility functions* U_t *such that this data is consistent with Pareto optimal demand (or the Collective Rationality model) if and only if:*

$$\forall r = 1,2 \quad C^r = Y^r + w_a^r \ell_a^r + w_b^r \ell_b^r \quad AND$$

For some $r, s = 1, 2$, $r \neq s$, *either:*

$$\textbf{I} \quad [C^s + w_a^r L_a^s > w_a^r L_a^r \quad AND \quad C^s + w_b^r L_b^s > w_b^r L_b^r \quad AND \quad C^s - C^r + w_a^r (L_a^s - L_a^r) + w_b^r (L_b^s - L_b^r) > 0]$$

or:

$$\textbf{II} \quad [C^s + w_a^r L_a^s > w_a^r L_a^r \quad AND \quad C^r + w_b^s L_b^r > w_b^s L_b^s]$$

Proof: See Appendix.

For example, suppose we observe 2 years of a household's labor supply behavior, hourly wages, and non-labor income.[14] If we assume the household follows a balanced budget in spending, we can solve for consumption levels in both years, and apply the restrictions above to test for Pareto optimal behavior within the household.

Suppose we observe the following data:[15]

	Year 1	Year 2
Husband's wage	5.00	19.00
Wife's wage	3.00	16.00
Non-labor income	10,270.00	0.00
Husband's hours worked	2000	500
Wife's hours worked	2000	250

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[14] Data of this sort is available, for example, from the Bureau of Labor Statistics' National Longitudinal Surveys

[15] In this example inflation will be ignored; in practice, one could use general data on prices levels, such as CPI data, to normalize the prices.

This data is not consistent with the Collective Rationality conditions as given above. In other words, this behavior is not consistent with Pareto optimal behavior within the household. On the other hand, this data is:

	Year 1	Year 2
Husband's wage	5.00	19.00
Wife's wage	3.00	16.00
Non-labor income	10,270.00	0.00
Husband's hours worked	2000	2000
Wife's hours worked	2000	2000

Note that the difference between these examples is that in the first case, both husband and wife face a large wage increase, but cut back their hours worked, while in the second case they face the same wage increase but remain full-time workers.

Empirical work on intra-household allocation has been performed with parametric specifications of preferences; for work specific to the Chiappori model, see Browning et al. (1996). Rejection of any model in a parametric framework could mean rejection of the fundamental model, or rejection of the parametric specification. The tests described above do not require any specification of preferences, and so lead to the broadest possible tests of the model. Their application should provide a useful counterpart to the existing empirical work on intra-household allocation.[16]

5 Conclusion

This paper has developed formal tests of efficiency of public good provision based on revealed preference theory. In contrast to existing methods of testing, the tests developed here are nonparametric: for example, the assumptions about preferences are that they can be represented by continuous, strictly monotonic, strictly quasiconcave, utility functions. Thus these tests are extremely broad; rejection of Pareto optimality is not a rejection of a particular parametric specification of preferences. As such they should provide a useful counterpart to existing work on tests of Pareto efficient public good provision.

Additionally, using the technique of quantifier elimination, we are able to derive tests that require mostly aggregate level data — no individual characteristics are required to be observed except after-tax income. Thus there should be a broad variety of situations where it is feasible to obtain the data required to test whether an optimal amount of public goods has been provided.

The Tarski-Seidenberg theorem tells us that it is theoretically possible to derive testable restrictions for an economy with any finite number of consumers or observations. The lack of practical quantifier elimination algorithms makes this difficult,

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[16] For work applying similar types of nonparametric tests of household behavior to household consumption and labor supply data, see Snyder (1997).

however. The development of quantifier elimination algorithms that are specifically designed for use with economic models — particularly models centered around simple individual utility maximization and profit maximization (or cost minimization) behavior — appears to be a potentially useful area of research in computational economics.

The methods outlined here are also capable of accommodating more complex models of public good provision. The theory could be extended to derive nonparametric testable restrictions of more general models of economies with externalities, local public goods and club goods. Thus the techniques outlined in this paper should provide a strong theoretical basis for nonparametric hypothesis tests and specification tests in a broad variety of situations where private goods, public goods, and goods that are some mixture of the two are present.

Appendix

Proof of Lemma 1 We must prove that if and only if the Pareto optimal public good demand restrictions are satisfied, there exists a solution of the following system of inequalities over $\langle q_t^r, y_t^r \rangle$:

$$\exists r, s, \quad r \neq s \quad \text{s.t.} \quad q_t^r x^r + p^r y_t^r < q_t^r x^s + p^r y_t^s \quad t = a, b \tag{1}$$

$$p^r y_t^r = I_t^r \quad t = a, b; \quad r = 1, 2 \tag{2}$$

$$y_a^r + y_b^r = y^r \quad r = 1, 2 \tag{3}$$

$$q_a^r + q_b^r = q^r \quad r = 1, 2 \tag{4}$$

Condition 1 means that WARP is satisfied for each consumer. Each consumer has two ways of satisfying WARP (period 1 consumption not revealed preferred to period 2 consumption or period 2 consumption not revealed preferred to period 1 consumption), thus there are four ways (not mutually exclusive) for both consumers to satisfy WARP. Thus conditions 1-4 are equivalent to the disjunction of four linear programs, with each one of these programs corresponding to one of the four different ways in which WARP can be satisfied. We will prove the lemma by showing that if and only if the public good consumption restrictions are satisfied will there exist a solution to at least one of these programs.

Case 1: Here we prove that if and only if condition **I** holds, then WARP can be satisfied by individual a's period r consumption not being revealed preferred to his period s consumption and individual b's period r consumption not being revealed preferred to his period s consumption for some r, s, $r \neq s$.

Necessary: Suppose $\{q_a^r, y_a^r\}$, $r = 1, 2$, satisfies condition 1-4 in this way. Then it must be true that $q_a^r x^r + p^r y_a^r < q_a^r x^s + p^r y_a^s \iff I_a^r < q_a^r (x^s - x^r) +$

$p^r y^s_a$. This implies $I^r_a < E^{rs}_a$. Similarly it must be true that $q^r_b x^r + p^r y^r_b < q^r_b x^s + p^r y^s_b \iff I^r_b < q^r_b(x^s - x^r) + p^r y^s_b$. This implies $I^r_b < E^{rs}_b$; also this condition can be rewritten $p^r y^r - I^r_a < q^r(x^s - x^r) - q^r_a(x^s - x^r) + p^r y^s - p^r y^s_a \iff q^r_a(x^s - x^r) + p^r y^s_a < I^r_a + q^r(x^s - x^r) + p^r(y^s - y^r)$. This is only possible if $q^r(x^s - x^r) + p^r(y^s - y^r) > 0$.

Sufficient: Suppose condition **I** holds. Then since $I^r_a < E^{rs}_a$, there exists some feasible $\{q^r_a, y^s_a\}$ such that $I^r_a < q^r_a(x^s - x^r) + p^r y^s_a$. Rewriting this condition, we get $q^r_a x^r + I^r_a < q^r_a x^s + p^r y^s_a$, which states that a's period r consumption is not revealed preferred to her period s consumption. Similarly, there also exists some feasible $\{q^r_b, y^s_b\}$ such that b's period r consumption is not revealed preferred to her period s consumption. To see that there exists feasible prices and consumptions such that conditions 3-4 are also satisfied (or that can satisfy WARP for each consumer simultaneously), note that $q^r(x^s - x^r) + p^r(y^s - y^r) > 0$ implies $[I^r_a, I^r_a + q^r(x^s - x^r) + p^r(y^s - y^r)]$ is a non-empty interval. Let e^{rs}_a be the value of the minimization problem corresponding to E^{rs}_a. The value of the expression $q^r_a(x^s - x^r) + p^r y^s_a$ ranges continuously in the interval $[e^{rs}_a, E^{rs}_a]$ as we vary y^s_a from 0 to $y^s$and as we vary q^r_a from 0 to q^r. Since $I^r_a < E^{rs}_a$, these intervals have a non-empty intersection. Thus there exist feasible $\{\widehat{q}^s_a, \widehat{y}^s_a\}$ such that $I^r_a < \widehat{q}^r_a(x^s - x^r) + p^r \widehat{y}^s_a$ (satisfies WARP for consumer a) and $\widehat{q}^r_a(x^s - x^r) + p^r \widehat{y}^s_a < I^r_a + q^r(x^s - x^r) + p^r(y^s - y^r)$ (satisfies WARP for consumer b).

Case 2: Here we prove that if and only if condition **II** holds, then WARP can be satisfied by individual a's period r consumption not being revealed preferred to his period s consumption and individual b's period s consumption not being revealed preferred to his period r consumption for some r, s, $r \neq s$.

Necessary: Suppose $\{q^r_a, y^r_a\}$, $r = 1, 2$, satisfies condition 1-4 in this way. Then it must be true that $q^r_a(x^s - x^r) + p^r y^s_a > I^r_a$. This implies $E^{rs}_a > I^r_a$. It must also be true that $q^s_b(x^r - x^s) + p^s y^r_b > I^s_b$. This implies $E^{sr}_b > I^s_b$.

Sufficient: Suppose condition **II** holds. Then $E^{rs}_a > I^r_a$ implies there exist some feasible $\{q^r_a, y^r_a\}$ such that $q^r_a(x^s - x^r) + p^r y^s_a > I^r_a$, or $q^r_a x^s + p^r y^s_a > q^r_a x^r + p^r y^r_a$. Similarly, $E^{sr}_b > I^s_b$ implies there exist some feasible $\{q^s_b, y^s_b\}$ such that $q^s_b x^r + p^s y^r_b > q^s_b x^s + p^s y^s_b$. Then conditions 1 and 2 are satisfied; to see that there exists $\{q^r_a, y^r_a\}$ such that conditions 3-4 are also satisfied (or that can satisfy WARP for each consumer), note that any values of q^s_a, y^r_a will satisfy WARP for consumer a, and any values of q^r_b, y^s_b will satisfy WARP for consumer b. Thus choose q^s_b, y^r_b so that WARP is satisfied for consumer b, and q^r_a, y^s_a so that WARP is satisfied for consumer a, and then choose values for $q^r_b, q^s_a, y^r_a, y^s_b$ that satisfy conditions 3-4.

Numerical examples proving that the Pareto optimality hypothesis is

non-vacuous: The following examples prove that Lemma 1 provides non-vacuous testable restrictions that can neither be always satisfied nor never satisfied for the case when there is one public good and one private good.

Example of an economy that satisfies the restrictions:
(data given in the order $D^r = \langle x^r, y^r, q^r, p^r, I_a^r, I_b^r \rangle$)

$D^1 = \langle 1, 10, 1, 2, 6, 14 \rangle$

$D^2 = \langle 9, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$

Example of an economy that does not satisfy the restrictions:

$D^1 = \langle 1, 10, 1, 2, 10, 10 \rangle$

$D^2 = \langle 9, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$

Proof of Corollary 2 This follows almost directly from Lemma 1, with some different assumptions about the observable variables. We cannot observe individual after-tax or after-contribution incomes within the household; however we do observe private good "expenditures" through observing wages and labor/leisure choices. A more serious difference is we no longer observe the public good levels. Aggregate (household) consumption, C, is not a public good, but individual consumptions, c_a and c_b, are. We cannot observe individual consumptions, so we cannot observe the amount of each public good provided in the household. We do observe aggregate consumption, however, and because consumption must be non-negative, this provides restrictions on the possible values of c_a and c_b. Define the following maximization problems:

$$E_t^{rs} = \max_{\nu,\mu,\delta,\gamma} \mu(\delta - \gamma) + \nu(C^s - C^r + \gamma - \delta) + w_t^r L_t^s \quad s.t. \quad \begin{array}{ll} 0 \leq \nu \leq 1 & 0 \leq \mu \leq 1 \\ 0 \leq \delta \leq C^s & 0 \leq \gamma \leq C^r \end{array}$$

These are the same maximization problems seen in Lemma 1 except these have two additional variables representing the individual consumption choices, making the problems bi-linear.

Applying Lemma 1, the testable restrictions are:

I $[E_a^{rs} > w_a^r L_a^r \text{ AND } E_b^{rs} > w_b^r L_b^r \text{ AND } C^s - C^r + w_a^r(L_a^s - L_a^r) + w_b^r(L_b^s - L_b^r) > 0]$
or:
II $[E_a^{rs} > w_a^r L_a^r \text{ AND } E_b^{sr} > w_b^s L_b^s]$

Solving the maximization problems, we find $E_t^{rs} = C^s + w_t^r L_t^s$.

References

S.N. Afriat. The construction of utility functions from expenditure data. *International Economic Review*, 8:67–77, 1967.

Donald J. Brown and Rosa L. Matzkin. Nonparametric estimation of smooth random utility functions. Mimeograph, Stanford University, 1994.

Donald J. Brown and Rosa L. Matzkin. Testable restrictions on the equilibrium manifold. *Econometrica*, 64:1249–62, 1996.

Martin Browning, François Bourguignon, Pierre-André Chiappori, and Valérie Lechene. Incomes and outcomes: A structural model of intra-household allocation. *Journal of Political Economy*, 102:1067–1096, 1996.

Pierre-André Chiappori. Rational household labor supply. *Econometrica*, 56:63–89, 1988.

Pierre-André Chiappori. Demand function for collective goods: Theory and application. *Annales Déconomie et de Statistique*, pages 27–42, 1990.

George B. Dantzig and B. Curtis Eaves. Fourier-motzkin elimination and its dual. *Journal of Combinatorial Theory*, 14:288–297, 1973.

Duncan K. Foley. Resource allocation and the public sector. *Yale Economic Essays*, pages 45–98, 1967.

Duncan K. Foley. Lindahl's solution and the core of an economy with public goods. *Econometrica*, 38:66–72, 1970.

Peter Hammond. Theoretical principles of economic policy: The public environment. Mimeograph, Stanford University, 1992.

G. Hanoch and M. Rothschild. Testing the assumptions of production theory: A nonparametric approach. *Journal of Political Economy*, 80:256–75, 1972.

Timothy J. Kehoe. Comparative statics. In Peter Newman John Eatwell, Murray Milgate, editor, *General equilibrium*, pages 76–83. W.W. Norton and Company, New York, 1987.

Jyoti Khanna. Cooperative versus noncooperative behavior: The case of agricultural research. *Review of Economics and Statistics*, 75:346–352, 1993.

A.P. Kirman and K.J. Koch. Market excess demand in exchange economies with identical preferences and collinear endowments. *Review of Economic Studies*, 53:457–463, 1986.

Alan P. Kirman. Whom or what does the representative individual represent. *Journal of Economic Perspectives*, 6:117–136, 1992.

Jean-Jacques Laffont. *Fundamentals of Public Economics.* The MIT Press, Cambridge, Mass, 1988.

Shelly Lundberg and Robert A. Pollak. Noncooperative bargaining models of marriage. *American Economic Review*, 84:132–137, 1994.

Marilyn Manser and Murray Brown. Marriage and household decision-making: a bargaining analysis. *International Economic Review*, 21:31–44, 1980.

Rosa L. Matzkin and Marcel K. Richter. Testing strictly concave rationality. *Journal* *of Economic Theory*, 53:287–303, 1991.

Marjorie B. McElroy and Mary Jean Horney. Nash-bargained household decisions: toward a generalization of the theory of demand. *International Economic Review*, 22:333–349, 1981.

Martin McGuire and Carl H. Groth, Jr. A method for identifying the public good allocation process within a group. *Quarterly Journal of Economics*, 100:915–934, 1985.

Jean-Claude Milleron. Theory of value with public goods: A survey article. *Journal of Economic Theory*, 5:419–477, 1972.

Todd Sandler and James C. Murdoch. Nash-Cournot or Lindahl behavior?: An empirical test for the NATO allies. *Quarterly Journal of Economics*, pages 875–894, 1990.

Susan K. Snyder. Testable restrictions of general equilibrium models. VPI Working paper E95-31, 1995.

Susan K. Snyder. Nonparametric tests of household behavior. VPI Working Paper E97-03, 1997.

Alfred Tarski. *A decision method for elementary algebra and geometry.* University of California Press, Berkeley, CA, second edition, 1951.

Hal R. Varian. The nonparametric approach to demand analysis. *Econometrica*, 50:945–972, 1982.

Hal R. Varian. The nonparametric approach to production analysis. *Econometrica*, 52:579–597, 1984.