Nonparametric testable restrictions of household behavior

Susan K. Snyder[1]
Department of Economics
Virginia Polytechnic Institute & State University
Blacksburg, VA 24061-0316
sksnyder@vt.edu

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Abstract

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This paper extends the analysis of testable restrictions of nonparametric equilibrium models to a model of household labor supply. We use semialgebraic theory to derive nonparametric testable restrictions of Pareto efficient bargaining behavior within a household. These tests are analogous in form to Samuelson's Weak Axiom of Revealed Preference (WARP) and are defined only over potentially observable variables: household-level data plus individual labor supplies. Thus without observing the allocation of consumption within the household, we can nonparametrically test whether there exist nonsatiated utility functions such that the household's behavior is consistent with Pareto efficiency. We apply these tests to data from the National Longitudinal Surveys on U.S. households and find that there do exist preferences that are consistent with Pareto efficiency for each household in the data set.

Keywords: household bargaining, revealed preference, quantifier elimination, Fourier-Motzkin elimination

JEL Classification: D11,D12

1 Introduction

Economists often treat the household as a single, utility-maximizing agent, regardless of the number of members who make up the household. There is certainly intuitive appeal to the idea that a household's members have common goals upon which they act; however, modeling households in this way is often inconsistent with the dominant model of behavior in economics, the model of the individual rational decisionmaker. One reason for the persistence of the unitary model of the household is that observing intra-household decisionmaking processes, allocations, or income divisions is generally difficult. More often data are available at the household level. Thus if we want to model households as collections of individually rational agents, we should ask what empirical implications for household behavior result from individual utility maximization.

This paper uses semialgebraic theory to derive nonparametric testable restrictions of Pareto efficient bargaining behavior within the household. These tests are in the form of a set of polynomial inequalities defined only over potentially observable variables: household-level data plus individual labor supplies. The derived tests are analogous in form to Samuelson's Weak Axiom of Revealed Preference (WARP), a nonparametric test of individual utility maximization. These testable restrictions are necessary and sufficient conditions; as such they are all the testable restrictions of the model given the data we assume are observable.

A previous approach to this problem is found in Chiappori (1988), who first presented non-vacuous testable restrictions on household-level data and individual labor supplies of Pareto efficient behavior within the household. These are necessary and sufficient tests that are in the form of finding whether a set of polynomial inequalities has a solution — if the program has a solution, then the data are consistent with the model; if not, the data are not consistent with the model.

A related problem is that of determining the empirical implications for aggregate demand data generated by individual utility maximization. Most results relating to this problem are negative, however; for example, well-known results in aggregation theory tell us that the aggregate demand of a group of rational decisionmakers has the same characteristics as the demand of one rational decisionmaker only under very strong restrictions (Shafer and Sonnenschein, 1982). One positive result relating to this problem is shown by Brown and Matzkin (1996), who use semialgebraic theory to find non-vacuous testable restrictions on discrete observations of the equilibrium manifold of an economy. Thus there are interesting empirical implications generated by competitive equilibrium behavior on aggregate-level data together with individual endowments or incomes. Another interpretation of their result is that individual utility maximization generates empirical implications on aggregate demand data together with data on individual incomes for members of a group or economy.

In this paper we merge these two approaches: using semialgebraic theory introduced in Brown and Matzkin (1996), we derive testable restrictions for Pareto efficient intra-household allocation. These tests are of a different form than, but are equivalent to, the linear programming tests derived in Chiappori (1988). Like WARP, these tests will be easy to apply in practice, and can be readily compared to and interpreted in terms of the nonparametric testable restrictions of other hypotheses of household behavior.

Empirical work on household or individual consumption is usually conducted with restrictive assumptions about the preferences of the decisionmakers. Generally, parametric specifications of the utility functions are used in order to derive testable propositions of the models. For example, the empirical work testing between the unitary model and models of Pareto efficient intra-household allocation has used parametric methods (for example, see Browning et al. (1994)). The validity of this work then depends in part on the validity of these assumptions about preferences.

The strength of nonparametric tests such as those presented in this paper is that they make very weak assumptions about preferences, essentially only that utility functions are nonsatiated. Additionally, if we have more than one observation of each household over time, we need make no assumptions about preferences across households. These tests could prove to be particularly useful as specification tests before one does more traditional econometric work on a dataset.

The outline of the paper is as follows. Section 2 gives a more detailed explanation of the collective rationality model. Section 3 derives the nonparametric testable restrictions of the model. Section 4 discusses how to use these tests to distinguish whether households behave as unitary actors or as a collective of rational individuals, with an application to data on U.S. households. The conclusion follows.

2 Collective Rationality Within the Household

The recognition that the unitary model of the household is often inconsistent with individual rationality, as well as the inability of the unitary model to meaningfully address questions concerning the distribution of income or consumption within the household, has led to the development of models of collective decisionmaking within the household. Using both cooperative and noncooperative bargaining theory, these models describe household behavior as the outcome of an explicit bargaining process between the members of the household (for example, see Manser and Brown (1980), McElroy and Horney (1981), Lundberg and Pollak (1994)). These models can be used to derive rich insights into intra-household allocation. However, the empirical propositions derived from these models often depend on intra-household data that can be difficult to observe. Thus it is difficult to test whether these collective models describe household behavior better than the unitary model. Empirical results suggest, however, the unitary model restrictions are often not satisfied regardless of the alternative model specified; see for example Schultz (1990), and Thomas (1990).

Chiappori (1988) develops a more general model of collective rationality in household behavior that can be tested with data on aggregate household behavior and individual labor supplies. The hypothesis is that the individuals within the household reach a Pareto efficient allocation. Thus instead of specifying a particular point on the contract curve as a function of individual threat points, as a Nash-bargaining model would, this model specifies only that the household be somewhere on this contract curve.

The model is as follows. A household is made up of two individuals, a and b. For $i = a, b$, each member can supply some amount of labor, ℓ_i, in a market outside the household. Let T represent the fixed amount of total time available to each a and b; $L_i = T - \ell_i$ defines leisure consumption for each individual (there is no household production). There is also a privately consumed good C; let c_i, a non-negative number, denote each member's consumption of the good. The price of the consumption good is normalized to one. Consumption and labor choices are made given wages, w_a, w_b, and non-labor household income Y.

Here we focus on the variant of the collective rationality model that assumes that household members each have preferences over only their own personal consumption; in Chiappori's terminology these are *egoistic* agents (the model can incorporate more general preferences).[1] Assume each agent has preferences representable by a nonsatiated utility function $U_i(L_i, c_i)$.

While the exact mechanism for determining household consumption is left unspecified, we can think of a Pareto efficient allocation as resulting from individual decisions subject to an agreement about the sharing of resources within the household. Let an *income-sharing rule* be some function G: $G(w_a, w_b, Y) = (y_a, y_b)$ such that $y_a + y_b = Y$. Then if the consumption choices for each individual i, $\{L_i, c_i\}$, are the solution of the following problem:

$$\max U_i(L_i, c_i) \quad \text{s.t.} \quad c_i \leq y_i + w_i \ell_i$$

the resulting allocation will be Pareto efficient given $\{U_a, U_b, G\}$ (Chiappori (1992)). The sharing rule is completely unspecified, and it may change over time. Also y_i is allowed to be negative, which would imply one member agrees to transfer some portion of their wage income to the other.

This model constitutes a quite natural alternative to the basic unitary model of household behavior. Instead of thinking of the household maximizing one utility function subject to a budget constraint, we think of the household as being composed of two individuals, each maximizing their own utility function subject to their own budget constraint, with individual "incomes" summing up to household income.

Using parametric methods, Browning et al. (1994) estimate the collective rationality model for data on Canadian family expenditures, and do not reject the collective rationality restrictions. Additionally, they find that the restrictions implied by the unitary model are rejected. While the validity of these results depends in part

[1] In the non-egoistic model individual consumption becomes a public good within the household. Testable propositions of the non-egoistic model are derived using semialgebraic methods in Snyder (1999).

on the validity of the parametric assumptions made in estimation, the collective rationality model appears to be a promising alternative to the unitary model.

3 Nonparametric testable restrictions

We will first derive the testable restrictions of this model with the assumption that we can observe data on consumption choices and income at the individual level. We will then use those results to derive restrictions for when we cannot observe individual level data.

Suppose we could observe a series of observations on the behavior of a household both at the household level and at the individual level; that is, we observe household consumption, household non-labor income, wages, individual labor supplies, individual consumption, and individual shares of non-labor income, $\{C^r, Y^r, w_i^r, \ell_i^r, c_i^r, y_i^r\}$ for $i = a, b$, where i indicates a member of a household, and $r = 1, ..., R$, where r indicates a distinct time period. Can we say whether there exist any non-satiated utility functions U_i and a sharing rule G such that the collective rationality model could have generated this data? In other words, are there restrictions on the data that will tell us whether or not the data could have been generated by the collective rationality model, without making any further parametric specifications of preferences or the sharing rule?

In the collective rationality model each individual maximizes their own utility function subject to a budget constraint. A necessary and sufficient condition for a finite set of data on individual consumption and prices to be consistent with the maximization of some nonsatiated utility function is the satisfaction of the Generalized Axiom of Revealed Preference, a generalized version of WARP (Varian (1982)). It is clear that GARP is a necessary condition of utility maximization. It is the sufficient part of this theorem that is perhaps not obvious. This is not sufficient in the sense that the data can imply that consumers act as utility-maximizers — we can never know that for sure — but sufficient in the sense that the data impliy that there exists a nonsatiated utility function such that the consumer could be maximizing utility with her choices.[2] Given that we cannot observe utility functions, this is the strongest notion of sufficiency that the data can satisfy.

Thus the collective rationality model implies that each individual in the household must satisfy GARP. Together with some aggregation conditions, this gives us all the empirical implications of the collective rationality model.

The following theorem and corollary are restatements of Chiappori's (1988) Proposition 2:[3]

[2]Further, if there exists a nonsatiated utility function that could have generated the data, there exists a concave utility function that could have generated the data, see Afriat (1967).

[3]To eliminate some uninteresting cases of restrictions, we assume wages are nonzero, and we also assume (here and in theorem 2) that $\{Y^1, w_a^1, w_b^1\} \neq \{Y^2, w_a^2, w_b^2\}$, that is, there is some change in at least one of the variables exogenous to the household.

Theorem 1 *Let data for one household,* $\{C^r, Y^r, w_i^r, \ell_i^r, c_i^r, y_i^r\}$ *for* $i = a, b$, *and* $r = 1, 2$, *be given. Then there exist strictly monotonic, strictly concave utility functions* $\{U_i\}_{i=a,b}$ *and income-sharing rule* $\{G\}$ *such that the data are consistent with a Pareto optimal allocation within the household* $\langle U_a(L_a, c_a), U_b(L_b, c_b)\rangle$ *iff:*

1. *Household aggregation conditions satisfied (for* $r = 1, 2$*):*

$$c_a^r + c_b^r = C^r$$
$$y_a^r + y_b^r = Y^r$$

2. *Individual budget constraints satisfied:*

$$c_i^r = y_i^r + w_i^r \ell_i^r \quad \text{for } i = a, b;\ r = 1, 2$$

3. *Individuals satisfy Strong Axiom of Revealed Preference (SARP)*[4]*:*

$$c_i^2 + w_i^1 L_i^2 > c_i^1 + w_i^1 L_i^1 \qquad \text{OR} \qquad c_i^1 + w_i^2 L_i^1 > c_i^2 + w_i^2 L_i^2 \quad \text{for } i = a, b$$

4. *Feasibility satisfied:*

$$c_a^r \geq 0, \quad c_b^r \geq 0$$

This theorem provides a restatement of the collective rationality model in the form of a finite set of polynomial inequalities defined over a finite set of variables. As such, it provides the testable restrictions of the collective rationality model, given we could observe both household and individual level behavior. If data from a given household satisfy these inequalities, then there exist utility functions and income-sharing rules such that the data are consistent with collective rationality. If data do not satisfy these inequalities, then there are no nonsatiated utility functions (with single-valued demand) such that the data could have been generated by the collective rationality model.

We do not expect it is generally possible to observe all these variables, however, particularly the intra-household income sharing or individual consumption. Corollary 1, which assumes that $\{c_i^r, y_i^r\}$, the individual consumptions and the assigned individual non-labor incomes, are unobserved, while $\{C^r, Y^r, w_i^r, \ell_i^r\}$, the household level consumptions and non-labor incomes, individual wages, and individual leisure/labor supplies, are observed, follows directly from theorem 1.

[4]Using the Strong Axiom of Revealed Preference instead of the Generalized Axiom greatly simplifies the resulting tests and simply rules out data that could have been generated only by utility functions that generate non-single-valued demands. If the data satisfy SARP, we can construct strictly monotone, strictly concave utility functions to generate the data, see Matzkin and Richter (1991).

Corollary 1 *Let* $\{C^r, Y^r, w_i^r, \ell_i^r\}$ *for* $i = a, b$, *and* $r = 1, 2$ *be given. Then there exist strictly monotonic, strictly concave utility functions* $\{U_i\}_{i=a,b}$ *and an income-sharing rule* $\{G\}$ *such that the data are consistent with a Pareto optimal allocation within the household* $\langle U_a(L_a, c_a), U_b(L_b, c_b)\rangle$ *iff there exist numbers* $\{c_i^r, y_i^r\}$, $c_i^r \geq 0$, *such that the conditions of Theorem 1 are satisfied.*

This corollary provides nonparametric testable restrictions of the Collective Rationality model. The tests are of the following form: if there exist real solutions to the linear programs described in corollary 1, then the data are consistent with the model. If there do not exist real solutions to those programs, then the data are not consistent with the model (Chiappori, 1988). Nonparametric tests of this form have been developed extensively in Diewert and Parkan (1985), for example.

Note also that theorem 1 and corollary 1 present the collective rationality model as a finite set of polynomial inequalities in observed and unobserved variables. Thus the structure of the complete set of testable restrictions of this model is semialgebraic. This results not from any restrictions on preferences; it is the finiteness of data that limits the testable restrictions of utility maximization to be of polynomial form.

Brown and Matzkin (1996) use semialgebraic theory to examine the empirical implications of the competitive equilibrium model. It can be shown that the complete set of testable propositions of the competitive equilibrium model over finite data can be written as a finite union of sets of polynomial inequalities; i.e., they form a semialgebraic set. Thus we can use the properties of semialgebraic geometry to address many empirical issues of general equilibrium. The complete set of testable propositions of the collective rationality model also form a semialgebraic set, so we can use semialgebraic theory to address empirical implications of this model as well.

Most importantly, semialgebraic theory provides finite-time methods for deriving testable propositions over a particular data structure. These testable propositions will be in the form of a finite union of sets of polynomial inequalities in the data (Mishra, 1993). In other words, given the data we believe to be observable, even if these data do not include many of the model's key variables, we have methods of systematically deriving all of the model's empirical propositions. These methods to derive the restrictions may prove infeasible in large problems (Van Den Dries, 1988); if so, we at least a) know the restrictions will be in the form of a semialgebraic set, and b) can address whether these restrictions are non-vacuous, which will often be of particular importance when key variables of the model are unobservable.

The derivation of testable propositions over a subset of a model's variables is accomplished through the technique of quantifier elimination. Quantifier elimination is the process of eliminating "quantified" variables — in our case, the unobservable variables are quantified within the model in that they appear in conjunction with the existential quantifier ("there exists").

There are three possible outcomes when quantifier elimination is applied to a system. One possibility is the equivalent system reduces to $1 \equiv 0$, meaning it is impossible to observe data consistent with the model; the theory is empirically inconsistent. Another possibility is the system reduces to $1 \equiv 1$, meaning it is impossible to observe data not consistent with the model; the theory imposes no testable restrictions on the set of observable variables. The third possibility is the system reduces to a finite set of polynomial inequalities involving the observable variables. It is possible to observe data that satisfy these inequalities, and it is possible to observe data that do not satisfy these inequalities. In this case we say testable restrictions of the model exist, given the set of observable variables (Brown and Matzkin, 1996).

It is straightforward to show that this system describing the collective rationality model satisfies empirical consistency; that is, we can find data that are consistent with the model. We can also show that there exist data that are not consistent with the model. Chiappori (1988) provides an example of three observations of data such that the revealed preference axioms cannot be satisfied for each individual; below we show an example of two observations of data such that the conditions are not satisfied. Thus there do exist non-vacuous testable restrictions of the collective rationality model over household-level data together with individual wages and labor supplies. If we apply quantifier elimination to the system described in corollary 1, we will derive these testable restrictions. These restrictions will be in the form of checking a set of polynomial inequalities defined directly over observable variables.

Because this system is linear, one can use Fourier-Motzkin elimination to eliminate the unobserved variables.[5] We apply this technique for two observations of data:

Theorem 2 *Let $\langle w_a^r, w_b^r, \ell_a^r, \ell_b^r, C^r, Y^r\rangle$ for $r = 1,2$ be given. Then there exist strictly monotonic, strictly concave utility functions $\{U_i\}_{i=a,b}$ and income-sharing rules $\{G^r\}$ such that the data are consistent with a Pareto optimal allocation within the household $\langle U_a(L_a, c_a), U_b(L_b, c_b)\rangle$ iff the collective rationality nonparametric restrictions are satisfied.*

Collective Rationality Model Nonparametric Restrictions

$$\forall r = 1,2 \quad C^r = Y^r + w_a^r \ell_a^r + w_b^r \ell_b^r \quad \text{AND}$$
$$\exists r,s = 1,2, r \neq s \text{ s.t.} \quad \{(C^s + w_a^r L_a^s > w_a^r L_a^r \quad \text{and} \quad C^r + w_b^s L_b^r > w_b^s L_b^s) \quad \text{OR}$$
$$(C^s + w_a^r L_a^s + w_b^r L_b^s > C^r + w_a^r L_a^r + w_b^r L_b^r \quad \text{and}$$
$$C^s + w_a^r L_a^s > w_a^r L_a^r \quad \text{and} \quad C^s + w_b^r L_b^s > w_b^r L_b^r)\}$$

Proof: We used Fourier-Motzkin elimination to rewrite the equilibrium conditions of the model in terms of only the observable variables. See Appendix.

[5]Fourier-Motzkin elimination (sometimes called Fourier elimination) is a technique similar to Gaussian elimination. See Dantzig and Eaves (1973).

Compare the collective rationality restrictions to the testable restrictions of the unitary model:

Unitary Model Nonparametric Restrictions

$$\forall r = 1,2 \quad C^r = Y^r + w_a^r \ell_a^r + w_b^r \ell_b^r \quad \text{AND}$$
$$\exists r, s = 1,2, r \neq s \text{ s.t.} \quad C^s + w_a^r L_a^s + w_b^r L_b^s > C^r + w_a^r L_a^r + w_b^r L_b^r$$

The household budget constraint must be satisfied under both models. An additional restriction in the unitary model is that the household consumption choices must satisfy the Strong Axiom of Revealed Preference (SARP). As one would expect, the collective rationality model does not imply SARP and is not implied by SARP. Thus it is possible to observe data that are consistent with one model and not the other, both models, or neither model.

The collective rationality restrictions result from the definition of bounds on consumption and labor supply data such that it is possible for each individual in the household to be satisfying the axiom of revealed preference. The non-vacuousness of the restrictions comes essentially from the non-negativity restrictions on individual consumption. Given this level of observability, it is always possible for each individual to be maximizing utility given any data; it is not always possible, however, for each individual to be maximizing utility given that the other individual in the household is maximizing utility as well.

Figure one illustrates a situation where the restrictions will not be satisfied. Because the consumption good can be traded within the household, it is useful to use an Edgeworth box to model the problem each period, keeping in mind that leisure is not tradeable within the household. The first picture represents the situation for consumer a. Each period consumer a faces a utility maximization problem where he chooses between leisure and consumption. We do not know how income is divided up within the household, but we do know the slope of the budget constraint that a faces (because we observe the normalized wages), and we know that the budget constraint in each period r must pass through some point (c_a^r, L_a^r), where L_a^r, leisure, is observed. Thus we know the range of possible budget constraints in each period, from highest level of income to lowest level of income. This range of budget constraints, from B_L to B_H, is represented on the graph each period.

Note that for any possible level of income, consumer $a's$ second period bundle is always revealed preferred to his first period bundle. Thus in order to satisfy SARP, income in the first period must be such that his first period consumption is not revealed preferred to his second period consumption. A budget constraint below $\hat{B}_a$ is necessary. In other words, a must consume less than $\hat{c}_a^1$ of the consumption good in order for SARP to be satisfied.

Suppose b faces the situation in the second graph. Leisure for consumer b replaces $a's$ leisure on the x axis, while the consumption good remains on the y axis. With these wages, again we have a situation where consumer $b's$ second period bundle is always revealed preferred to her first period bundle. Thus in order to satisfy SARP, income in the first period must be such that her first period is not revealed preferred to the second period bundle. Again we can find an upper bound on income such that this is possible. But in order to ensure that a satisfies SARP, it must be the case that b must consume at least $\hat{c}_b^1 = C^1 - \hat{c}_a^1$ of the consumption good. This puts a lower bound on $b's$ budget constraint, $\hat{B}_b$. In the situation here, this lower bound is *above* the upper bound on income that will satisfy SARP. Thus while each consumer could satisfy SARP, they cannot both satisfy SARP at the same time. These data fail the collective rationality restrictions.

4 Implementation and application

The collective rationality testable restrictions derived above are similar in form to revealed preference tests of individual utility maximization. They require little data to implement and are straightforward to apply.

If the data satisfy the restrictions, one could interpret this as the satisfaction of a specification test.[6] That is, if the data satisfy the collective rationality restrictions, then one could be more confident about doing further empirical work, making more restrictive assumptions about functional form, for example, and estimating parameters such as the sharing rule within the household. Note that these restrictions are necessary and sufficient conditions on the data; thus if these restrictions were satisfied by a particular dataset, there exist no other restrictions on that dataset that would imply the collective rationality model was not satisfied.

If the data do not satisfy the restrictions, it is not clear what the interpretation should be. As in other revealed preference tests, we have not allowed for any stochastic elements in the model or the data. Varian (1985, 1990) addresses the problem that there is likely to be some measurement error in the data that could cause nonparametric tests to lead to rejection. These papers address the question of how to judge how "close" the conditions are to being satisfied. Alternatively, one could posit that there is some stochastic randomness in preferences, as in Brown and Matzkin (1995), who discuss the nonparametric approach to testing when utility depends linearly on a random variable ϵ.

4.1 Application to data on U.S. households

We will apply these tests to data on U.S. households to determine whether these households appear to be behaving consistently with the collective rationality model, the unitary model, both, or neither. The data we use are from the National Longitudinal Surveys (NLS), sponsored by the Bureau of Labor Statistics. These surveys gathered information on the labor market activities and other income sources of

[6] See Diewert and Parkan (1983).

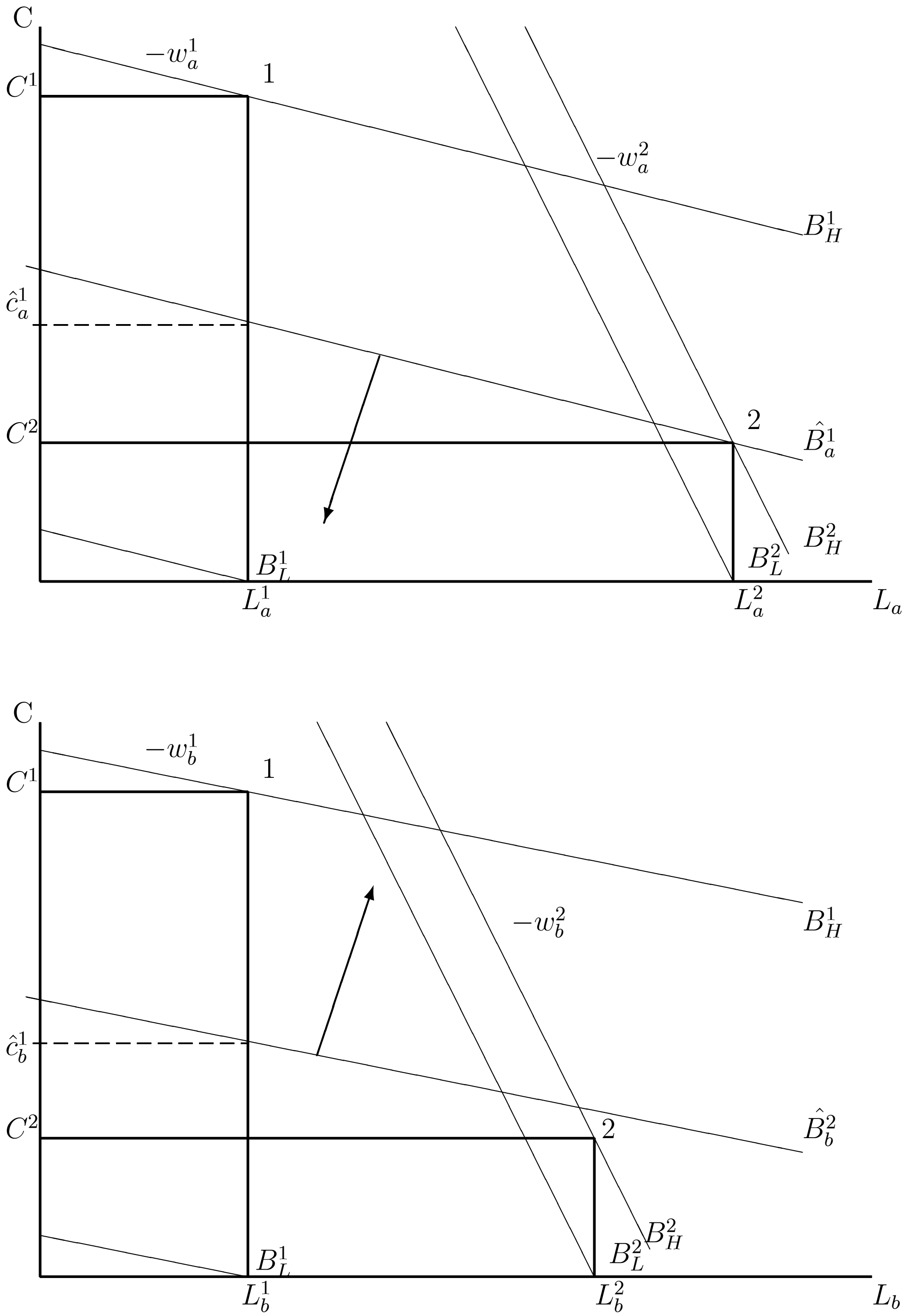

Figure 1: Data that do not satisfy the restrictions

selected American men and women over a number of years. Horney and McElroy (1988) use NLS data to estimate a (parametrized) Nash-bargaining model of household behavior, and found some evidence that this model performed better than the unitary model.

We use a subsample from the 1969 and 1971 National Longitudinal Survey of Men (the men were between ages 45-59 in 1966).[7] The subsample used only observations of families where: both spouses worked more than zero hours each year, there were no children, the family received no farm income, total household income was positive, and there were no missing observations for the variables we used in the tests. We make no claim that these households are representative of U.S. households in general; for example, these households contain workers that are older than average, and they have a higher percentage of women working outside the home than average.

Both the collective rationality nonparametric restrictions and the unitary restrictions are defined over two observations of data on household consumption, household non-labor income, wages, and individual labor supplies. We do not have data on consumption expenditures, so we have assumed that household budget constraints were satisfied so that household consumption expenditures equaled total income. The NLS provide data on many sources of non-labor income. Wages for each spouse were computed from data on hours worked and income from wages and salary. We normalized consumption expenditure and wages by each year's CPI-U price level. Simple statistics and more details about the variables used are given in the Data Appendix.

The results of the nonparametric tests of the collective rationality model and the unitary model are reported in Table 1. About 98% of the 243 households satisfied the restrictions for both models, and all households satisfied the Collective Rationality restrictions. Thus for every household in the sample, there exist individual non-satiated utility functions and an intra-household sharing rule such that household consumption and labor supply behavior is consistent with Pareto efficiency.

Table 1: **Results of Nonparametric tests: 243 households**

	Unitary Model Satisfied	Unitary Model Not Satisfied
Collective Rationality Satisfied	237	6
Collective Rationality Not Satisfied	0	0

The fact that the sample's households are all consistent with collective rationality is perhaps not surprising. We are essentially testing individually rational behavior with mostly aggregate-level (household-level) data. Also, the parametric estimations of the collective rationality model for other datasets have generally led to good fits;

[7] We replicated this approach for the 1969 and 1971 NLS Women dataset also and found very similar results.

when we weaken the maintained hypotheses made in these empirical studies by assuming no particular functional form, we should expect the results to be even more favorable.

A more fundamental problem with revealed preference tests is that sometimes consumers cannot violate revealed preference conditions no matter how they act. For example, if one household member had both his wage and his nonlabor income increase from year 1 to year 2, then he could not possibly violate SARP because his two budget lines do not cross — any bundle he could choose in year 2 would always be revealed preferred to any bundle he could choose in year 1, and any bundle he could choose in year 1 would never be revealed preferred to any bundle he could choose in year 2. In our problem, however, we assume individual nonlabor income is unobservable; thus there always exists an assignment of nonlabor income within the household such that at least one household member's budget lines cross. Thus given the exogenous data of wages and household-level nonlabor income, it is always theoretically possible to have behavior inconsistent with collective rationality.[8]

Additionally, the revealed preference conditions used in the collective rationality restrictions are knife-edge conditions; there is no stochastic element built in. Thus one might expect to see numerous violations in practice. Errors in reporting or recording the data, omitted variables, misspecification of the basic maximization problem faced by individuals (for example, the assumption that there is no in-house production) or random elements of preference or consumption behavior could all lead to violations of the restrictions.

Interestingly, while the collective rationality restrictions are always satisfied, the unitary restrictions are violated for some households. The number of households violating the unitary restrictions is too small to conclude that the collective rationality model fits the data better.[9]

Given that the data satisfy the collective rationality restrictions, one could proceed to doing further work with NLS data such as making functional form assumptions and estimating sharing rules within the household. Or, one could continue to work in the nonparametric setting and test sharper hypotheses about preferences or the sharing rule within the household. Further, one can recover bounds on utility functions and sharing rules consistent with this data (Afriat, 1967).[10]

[8]This does not address the "power" of the collective rationality test, however, or under what conditions it is possible to see a violation of the collective rationality conditions as given. See Bronars (1987) and Manser and McDonald (1988) for different approaches to computing the power of revealed preference tests.

[9]We replicated these tests for the survey of mature women in the NLS and found very similar results: of 108 households, all satisfy the collective rationality tests and 4 out of 108 (4%) fail the unitary tests.

[10]For more discussion on recovering utility functions from revealed preference tests, see Varian (1982). Approximate bounds could be recovered even if the tests were not satisfied for all observations (Afriat, 1973).

5 Conclusion

This paper has used semialgebraic theory to derive nonparametric testable restrictions of the collective rationality model of household behavior. These restrictions are necessary and sufficient conditions for household-level data (including individual-level labor supplies) to be consistent with Pareto-efficient intra-household allocation. These nonparametric restrictions should provide a useful supplement to econometric tests that use assumptions about parametric forms; for example, they could be used as specification tests.

We have used these restrictions to test whether data from the NLS describe households that act as one utility maximizing agent or two utility maximizing agents who arrange a Pareto efficient intra-household allocation. We conclude there do exist preferences that are consistent with Pareto efficiency for all of the households in this data set. We also find 2% of the households are not behaving consistently with the unitary model. Further investigations could perhaps focus on how close these households were to satisfying the unitary restrictions, as suggested in Varian (1985), Varian (1990); if the fit is close, then we might conclude we cannot reject the unitary model. If so, this could be some evidence that the parametric tests in other empirical work are rejecting not the unitary model but rather the particular parametric specification that was used.

6 Appendix: Proof of theorem 2

Equilibrium in the collective rationality model is described by the following quantified set of polynomial equalities and inequalities, Γ.

Γ: There exist numbers $\{c_i^r, y_i^r\}$ such that:

$$c_a^r + c_b^r = C^r \qquad \text{for } r = 1,2 \qquad (1)$$

$$y_a^r + y_b^r = Y^r \qquad \text{for } r = 1,2 \qquad (2)$$

$$y_i^r + w_i^r \ell_i^r = c_i^r \qquad \text{for } r = 1,2;\ i = a,b \qquad (3)$$

$$c_a^2 + w_a^1 L_a^2 > c_a^1 + w_a^1 L_a^1 \quad \text{OR} \quad c_a^1 + w_a^2 L_a^1 > c_a^2 + w_a^2 L_a^2 \qquad (4)$$

$$c_b^2 + w_b^1 L_b^2 > c_b^1 + w_b^1 L_b^1 \quad \text{OR} \quad c_b^1 + w_b^2 L_b^1 > c_b^2 + w_b^2 L_b^2 \qquad (5)$$

$$c_i^r \geq 0 \qquad \text{for } r = 1,2;\ i = a,b \qquad (6)$$

To derive an equivalent non-quantified set of polynomial equalities and inequalities, the variables $\{c_i^r, y_i^r\}$ must be eliminated. First use the equalities to make some obvious substitutions. Equations (1) allows us to solve for c_b^r, and equations (3) allow us to solve for y_i^r. Substituting back into Equations (2), we get:

$$Y^r + w_a^r \ell_a^r + w_b^r \ell_b^r \;=\; C^r \quad \text{for } r = 1,2$$

These are simply the household budget constraint conditions. They are defined over observable variables, and so form testable restrictions of the model.

The following system Θ, together with the household budget constraints, is equivalent to the original system:

Θ: There exist numbers $\{c_a^r\}$ such that:

$$\begin{array}{rcl} c_a^2 + w_a^1 L_a^2 > c_a^1 + w_a^1 L_a^1 & \text{OR} & c_a^1 + w_a^2 L_a^1 > c_a^2 + w_a^2 L_a^2 \\ C^2 - c_a^2 + w_b^1 L_b^2 > C^1 - c_a^1 + w_b^1 L_b^1 & \text{OR} & C^1 - c_a^1 + w_b^2 L_b^1 > C^2 - c_a^2 + w_b^2 L_b^2 \\ 0 \le c_a^r \le C^r & & \text{for } r = 1,2 \end{array}$$

This system is equivalent to Φ, the disjunction of four sets of linear inequalities:

Φ: Φ_1 or Φ_2 or Φ_3 or Φ_4 is satisfied.

Φ_1: There exist numbers $\{c_a^r\}$ such that:

$$\begin{array}{rcl} c_a^2 + w_a^1 L_a^2 & > & c_a^1 + w_a^1 L_a^1 \\ C^2 - c_a^2 + w_b^1 L_b^2 & > & C^1 - c_a^1 + w_b^1 L_b^1 \\ 0 \le c_a^r \le C^r & & \text{for } r = 1,2 \end{array}$$

Φ_2: There exist numbers $\{c_a^r\}$ such that:

$$\begin{array}{rcl} c_a^2 + w_a^1 L_a^2 & > & c_a^1 + w_a^1 L_a^1 \\ C^1 - c_a^1 + w_b^2 L_b^1 & > & C^2 - c_a^2 + w_b^2 L_b^2 \\ 0 \le c_a^r \le C^r & & \text{for } r = 1,2 \end{array}$$

Φ_3: There exist numbers $\{c_a^r\}$ such that:

$$\begin{array}{rcl} c_a^1 + w_a^2 L_a^1 & > & c_a^2 + w_a^2 L_a^2 \\ C^2 - c_a^2 + w_b^1 L_b^2 & > & C^1 - c_a^1 + w_b^1 L_b^1 \\ 0 \le c_a^r \le C^r & & \text{for } r = 1,2 \end{array}$$

Φ_4: There exist numbers $\{c_a^r\}$ such that:

$$\begin{array}{rcl} c_a^1 + w_a^2 L_a^1 & > & c_a^2 + w_a^2 L_a^2 \\ C^1 - c_a^1 + w_b^2 L_b^1 & > & C^2 - c_a^2 + w_b^2 L_b^2 \\ 0 \le c_a^r \le C^r & & \text{for } r = 1,2 \end{array}$$

Each Φ_i is a set of inequalities that is linear in the quantified variables so we can apply Fourier-Motzkin elimination to each set. If one Φ_i is satisfied, then Φ is satisfied. Note that Φ_1 and Φ_4 are the same sets of inequalities with reversed observations, and Φ_2 and Φ_3 are the same sets of inequalities with reversed observations.

6.1 Fourier-Motzkin elimination on Φ_1

First, we will eliminate c_a^1 from the Φ_1. Rewrite all the inequalities that involve c_a^1 with c_a^1 isolated:

$$\begin{aligned} c_a^1 &< c_a^2 + w_a^1 L_a^2 - w_a^1 L_a^1 \\ c_a^1 &> C^1 - C^2 + w_b^1 L_b^1 - w_b^1 L_b^2 + c_a^2 \\ c_a^1 &\geq 0 \\ c_a^1 &\leq C^1 \end{aligned}$$

There exists a c_a^1 such that that system is satisfied if and only if:

$$\begin{aligned} C^1 - C^2 + w_b^1 L_b^1 - w_b^1 L_b^2 + c_a^2 &< c_a^2 + w_a^1 L_a^2 - w_a^1 L_a^1 \\ 0 &< c_a^2 + w_a^1 L_a^2 - w_a^1 L_a^1 \\ C^1 - C^2 + w_b^1 L_b^1 - w_b^1 L_b^2 + c_a^2 &< C^1 \end{aligned}$$

The first inequality can be rewritten

$$C^2 + w_a^1 L_a^2 + w_b^1 L_b^2 \quad > \quad C^1 + w_a^1 L_a^1 + w_b^1 L_b^1$$

There are no unobservables in this inequality, so it is one of the testable restrictions of this system. Now to eliminate c_a^2 from the rest of the system, isolate the c_a^2:

$$\begin{aligned} c_a^2 &< w_a^1 (L_a^1 - L_a^2) \\ c_a^2 &< C^2 + w_b^1 (L_b^2 - L_b^1) \\ c_a^2 &\geq 0 \\ c_a^2 &\leq C^2 \end{aligned}$$

There exists a c_a^2 such that that system is satisfied if and only if:

$$\begin{aligned} w_a^1(L_a^1 - L_a^2) &< C^2 + w_b^1(L_b^2 - L_b^1) \\ w_a^1(L_a^1 - L_a^2) &< C^2 \\ 0 &< C^2 + w_b^1(L_b^2 - L_b^1) \end{aligned}$$

Note that the first inequality is implied by the first testable restriction. The system Φ_1 can be satisfied if and only if:

$$\begin{aligned} C^2 + w_a^1 L_a^2 + w_b^1 L_b^2 &> C^1 + w_a^1 L_a^1 + w_b^1 L_b^1 \\ C^2 + w_a^1 L_a^2 &> w_a^1 L_a^1 \\ C^2 + w_b^1 L_b^2 &> w_b^1 L_b^1 \end{aligned}$$

The testable restrictions of system Φ_4 are derived in a similar way. They are:

$$\begin{aligned} C^1 + w_a^2 L_a^1 + w_b^2 L_b^1 &> C^2 + w_a^2 L_a^2 + w_b^2 L_b^2 \\ C^1 + w_a^2 L_a^1 &> w_a^2 L_a^2 \\ C^1 + w_b^2 L_b^1 &> w_b^2 L_b^2 \end{aligned}$$

6.2 Fourier-Motzkin elimination on Φ_2

First, we will eliminate c_a^1 from the system. Rewrite all the inequalities that involve c_a^1 with c_a^1 isolated:

$$\begin{aligned} c_a^1 &< c_a^2 + w_a^1 L_a^2 - w_a^1 L_a^1 \\ c_a^1 &< C^1 - C^2 + w_b^2 L_b^1 - w_b^2 L_b^2 + c_a^2 \\ c_a^1 &\geq 0 \\ c_a^1 &\leq C^1 \end{aligned}$$

There exists a c_a^1 such that that system is satisfied if and only if:

$$\begin{aligned} 0 &< c_a^2 + w_a^1 L_a^2 - w_a^1 L_a^1 \\ 0 &< C^1 - C^2 + w_b^2 L_b^1 - w_b^2 L_b^2 + c_a^2 \end{aligned}$$

Now to eliminate c_a^2 from the rest of the system, isolate the c_a^2:

$$\begin{aligned} c_a^2 &> w_a^1(L_a^1 - L_a^2) \\ c_a^2 &> C^2 - C^1 + w_b^2 L_b^2 - w_b^2 L_b^1 \\ c_a^2 &\geq 0 \\ c_a^2 &\leq C^2 \end{aligned}$$

There exists a c_a^2 such that that system is satisfied if and only if:

$$\begin{aligned} w_a^1(L_a^1 - L_a^2) &< C^2 \\ C^2 - C^1 + w_b^2 L_b^2 - w_b^2 L_b^1 &< C^2 \end{aligned}$$

The system Φ_2 can be satisfied if and only if:

$$\begin{aligned} C^2 + w_a^1 L_a^2 &> w_a^1 L_a^1 \\ C^1 + w_b^2 L_b^1 &> w_b^2 L_b^2 \end{aligned}$$

The testable restrictions of Φ_3 are derived in a similar way. They are:

$$\begin{aligned} C^1 + w_a^2 L_a^1 &> w_a^2 L_a^2 \\ C^2 + w_b^1 L_b^2 &> w_b^1 L_b^1 \end{aligned}$$

7 Data Appendix

Table 2: Descriptive Statistics (n=243)

Variable	Mean	SD	Min.	Max.
Hours worked (yearly)[11]				
Husbands, 1968/69[12]	2079	454	350	3600
Wives, 1968/69	1730	592	80	2912
Husbands, 1970/71	2094	314	800	3600
Wives, 1970/71	1688	618	30	3120
Income from wages and salary				
Husbands, 1968/69	7570	3153	1000	18600
Wives, 1968/69	4007	2253	100	12000
Husbands, 1970/71	8586	4105	1200	32000
Wives, 1970/71	4636	2729	100	15000
Hourly Wages[13]				
Husbands, 1968/69	4.23	3.90	0.50	30.50
Wives, 1968/69	2.38	1.42	0.16	12.50
Husbands, 1970/71	4.14	2.01	0.48	16.00
Wives, 1970/71	3.15	4.60	0.19	66.67
Non-labor income[14]				
Non-labor income, 1968/69	460	1011	-469	7700
Non-labor income, 1970/71	709	1514	-930	13900
CPI-U price level, 1969	36.7			
CPI-U price level, 1971	40.5			

[11] All data required were not available in any two years so a mixture of 1968 and 1969 data are used to create the 1969 observations and a mixture of 1970 and 1971 data are used to create the 1971 observations.

[12] Yearly hours worked for husbands for both observations was computed by multiplying hours worked per week by 50.

[13] Computed from hours worked and income data.

[14] Sources are: Net income from business, partnership, rent; income from Bonds, Stocks and Mutual Funds; interest and dividends; unemployment compensation; veterans compensation/pension; workmen's compensation; aid to the disabled or blind; other disability payments; social security, both disability and other; welfare or public assistance; food stamps; government pensions; retirement pensions; and "other sources". In computing non-labor income, we set missing values for any of these variables to zero.

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