SOUTHEASTERN MASSACHUSETTS CONFERENCE MATH LEAGUE

Meet 3 – January 9, 2008

Round 1: Combinations, Permutations and Simple Probability

All answers must be in simplest exact form

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1. How many permutations of the "word" SMCML can be formed if all five of the letters are used?

2. How many positive integers less than 450 can be formed by using the digits 1, 2, 4, 5, 7 and 8, if no digit can be repeated in any number?

3. An urn contains 21 balls, identical except for their color. Four of the balls are blue, 8 are red and the rest are yellow. If two balls are removed from the urn at random, and without replacement, find the probability that both balls are the same color. Please express your answer as a fraction reduced to lowest terms.

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Round 2: Algebra 1 – Factoring and Rational Expressions

All answers must be in simplest exact form

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1. Write the following complex fraction as a single fraction, reduced to lowest terms (where $x \neq -1, 0, 1$):

$$\frac{x-1}{x-\frac{1}{x}}$$

2. Find the value of N so that $\frac{N}{x-5}+\frac{3}{x+4}=\frac{10x+13}{x^2-x-20}$ (where $x \neq -4, 5$).

3. Factor the following expression as the product of two simplified trinomials:

$$a^2+2bc-b^2-c^2$$

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Round 3: Pythagorean Theorem and Similarity

All answers must be in simplest exact form

The diagrams are not drawn to scale

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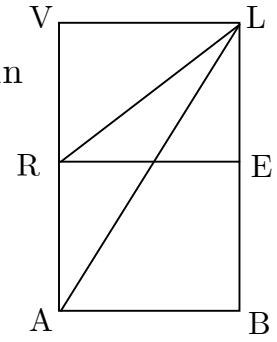
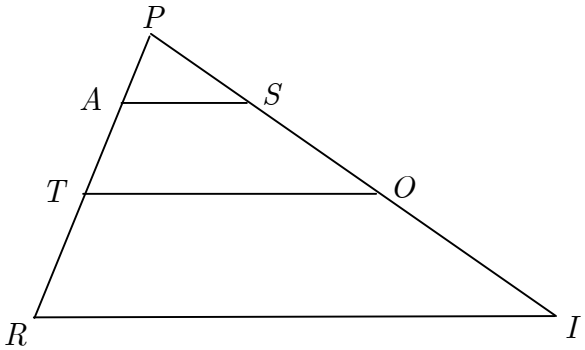
1. In the diagram at the right, VABL and RABE are rectangles with VL = 4, LR = 5, and VR = RA. Find the length of LA in simplest radical form.



2. Line segment BR is drawn parallel to side AD of triangle TAD with B on side TA and R on side TD. Also, segments RA and BD intersect at point Y. If BY = 8, RY = 7, and YD = 18, find the ratio $\frac{TR}{RD}$ in simplest form.

3. In the diagram below, $AS \parallel TO \parallel RI$ with $AS : RI = 1 : 5$ and $PS : SO = 2 : 3$. Find the ratio of the area of trapezoid $ATOS$ to the area of triangle PRI. Express your answer in simplest form.



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Meet 2 – January 9, 2008

Round 4: Algebra 2 – Exponents and Radicals

All answers must be in simplest exact form

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1. Find the value of n such that $(4(4(4^2))^2)^2 = 2^n$.

2. Let a, b, N, x and y be positive integers. Find the smallest possible value for the sum $N + x + y$ for which $\sqrt[3]{a^2b} \cdot \sqrt[4]{ab^2} = \sqrt[N]{a^x b^y}$.

3. Find all real numbers x such that $x + 2 = \sqrt{4 + x\sqrt{8 - x}}$.

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Round 5: Trigonometry – Reduction Formulas and Identities

All answers must be in simplest exact form

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1. If $\sin x = \frac{1}{3}$, find the value of $\frac{\cos x \cdot \tan x}{\csc x}$ as a fraction reduced to lowest terms.

2. If $\sin x + \cos x = \frac{1}{2}$, find the value of $\sin 2x$.

3. Simplify $\frac{\tan x - \cot x}{\tan x + \cot x}$ completely as an expression in terms of $\sin x$.

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Team Round: Matrices, Determinants and Algebra 1

Approved calculators are allowed.

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1. Find all values of x that make the following determinant equation true:

$$\begin{vmatrix} 2 & -1 & 5 \\ x & 3 & -4 \\ -2 & x & -3 \end{vmatrix} = 14$$

2. A cyclist rode one-third of the distance of a trip at 10 miles per hour, one-third at 9 miles per hour and the remainder at 8 miles per hour. If he had ridden half of the distance at 10 miles per hour and the other half at 8 miles per hour, the trip would have taken him one minute longer. Find the distance he traveled (in miles).

3. If $A = \begin{bmatrix} -1 & 2 \\ 1 & x \end{bmatrix}$, find all possible values of x such that the determinant of the matrix $A^2 + 2A$ is equal to -3.

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ANSWER SHEET – TEAM ROUND

1. _____

2. _____ miles

3. _____

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ANSWERS

Round 1 – Arithmetic

1. 60
2. 84
3. $\frac{1}{3}$ (only)

Round 2 – Algebra 1

1. $\frac{x}{x+1}$
2. 7
3. $(a+b-c)(a-b+c)$ (or equivalent)

Round 3 – Geometry

1. $2\sqrt{13}$ (only)
2. $\frac{4}{5} = 0.8 = 4:5$ (or equivalent)
3. $\frac{21}{100} = 0.21 = 21:100$ (or equivalent)

Round 4 – Algebra 2

1. 28
2. 33
3. -1 and 0 (need both, in any order)

Round 5 – Adv. Topics

1. $\frac{1}{9}$ (only)
2. $-\frac{3}{4} = -0.75$
3. $2\sin^2 x - 1$

Team Round

1. -2 and 1 (need both, in any order)
2. 36
3. -3 and 1 (need both, in any order)