

CALCULUS GROUP EXAM - MATH  
COMPETITION

Florida Gulf coast University

**Question 1**

Let there exist a real function  $f(x)$  such that  $\lim_{x \rightarrow c} f(x)$  does not exist for some real number  $c$ , but  $\lim_{x \rightarrow c} |f(x)| = L$  exist for some positive real number  $L$ . Which of the following statements must be true regarding the function  $f(x)$

- (A)  $\lim_{x \rightarrow c^+} f(x)$  and  $\lim_{x \rightarrow c^-} f(x)$  both exist
- (B)  $\lim_{x \rightarrow c} |f(x) - L| = 0$
- (C)  $f(c)$  does not exist
- (D)  $\lim_{\Delta x \rightarrow 0} \frac{|f(c + \Delta x) - f(c)|}{\Delta x}$  exists

• Answer:

A

**Question 2**

Use the three properties below to find the equation  $f(x)$

- $f(0) = 0$
- the graph of  $f$  has an **inflection point** at which the **tangent line is horizontal**, and
- $f''(x) = 2x + 2$  for all  $x$

- Answer:

$$\frac{x^3}{3} + x^2 + x$$

**Question 3**

Find  $f'(0)$  if

$$f(x) = \frac{\sqrt{1+2x} \sqrt[3]{1+3x}}{\sqrt[5]{1+5x} \sqrt[6]{1+6x} \sqrt[4]{1+4x}}$$

• Answer:

-1

**Question 4**

If  $(x^2 - y^2 - 1)^3 - y^3 = 0$  represents a curve, find the slope at a point  $(0, 1)$ .

• Answer:

0

**Question 5**

Let  $g$  be an infinitely differentiable with

$$g(3x - 1) = g'(2x + 6).$$

Given  $g(8) = 22$ , what is  $g''(44/3)$ ?

• Answer:

33

**Question 6**

Find an equation for the tangent line to the curve  $y = F(x)$  at the point  $P$  where  $x = 1$  and

$$F(x) = \int_1^{x^3} \frac{t^2 + 1}{t - 2} dt$$

• Answer:

$$y = -6(x - 1)$$

**Question 7**

If  $f$  satisfies  $f(x) + f\left(\frac{1}{1-x}\right) = \frac{1}{x}$  for  $x \neq 0, 1$ , find  $f(2)$ .

• Answer:

7/4

**Question 8**

Find the **sum** of constants  $A$ ,  $B$ , and  $C$  so that  $y = Ax^3 + Bx + C$  satisfies the equation

$$y''' + 2y'' - 3y' + y = x$$

• Answer:

4

**Question 9**

Evaluate  $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{1/n} + e^{2/n} + \dots + e^{n/n}]$

• Answer:

$$e - 1$$

**Question 10**

Suppose  $f$  is continuous and  $\frac{x^3}{3} \leq f(x) \leq 5$  for all  $x$  in the interval  $[0, 2]$ . Compute the values of largest  $A$  and smallest  $B$  such that

$$A \leq \int_0^2 f(x) dx \leq B$$

• Answer:

$$A=4/3, \quad B=10$$

**Question 11**

If  $f(x) = e^{x^2}$  and  $g(x) = \sqrt{\ln[(x^2 - 1)^2 + x]}$ , find

$$\frac{1}{\sqrt{f'(g(2))g'(2)}}$$

• Answer:

$$1/5$$

**Question 12**

List the letters of the functions that are differentiable at  $x = 0$ .

(A)  $y = |x^3 - 4x^2|$

(B)  $y = \sin^2 x$

(C)  $y = \tan |x|$

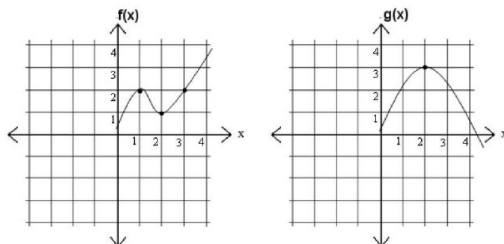
(D)  $y = |\sin(x)|$

• Answer:

A and B

**Question 13**

Graphs of  $f$  and  $g$  are shown. If  $h(x) = f(g(x))$ , which of the following statements could be true based on these graphs



$f$  has local extrema at  $x = 1$  and  $x = 2$ ,  $g$  has a local extrema at  $x = 2$

- (A)  $h(2) = 3$
- (B)  $h$  is increasing at  $x = 3$
- (C)  $h$  has a horizontal tangent line at  $x = 1$
- (D)  $h'$  is strictly positive at  $x = 4$

• Answer:

C

**Question 14**

Let  $f(x) = e^x$  and  $g(x) = f(x)^{f(f(x))}$ . Compute  $g'(0)$

• Answer:

e

**Question 15**

Evaluate

$$\lim_{x \rightarrow 0} \frac{\cos(x)e^x - 1}{x}$$

- Answer:

e