FGCU 6th Annual Math Competition 2008
Precalculus – Individual Exam

Find the domain of the rational function.

1) $f(x) = \frac{x + 2}{x^2 + 9}$

A) all real numbers
B) $\{x \mid x \neq -3, x \neq 3\}$
C) $\{x \mid x \neq -3, x \neq 3, x \neq -2\}$
D) $\{x \mid x \neq 0, x \neq -9\}$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

2) $e^{x + 7} = \frac{1}{e^{10}}$

A) –17
B) 3
C) –3
D) 17

Find the exact value of the expression, if possible. Do not use a calculator.

3) $\sin^{-1}\left[\sin\left(\frac{6\pi}{7}\right)\right]$

A) $\frac{7}{6\pi}$
B) $\frac{\pi}{7}$
C) $\frac{6\pi}{7}$
D) $\frac{7}{\pi}$

4) $\sin(\sin^{-1}\pi)$.

A) 2π
B) 0
C) π
D) Not Definted

Solve the equation on the interval [0, 2π).

5) $2\cos^2 x + \sin x - 2 = 0$

A) $\frac{\pi}{6}, \frac{5\pi}{6}$
B) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$
C) $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$
D) $\frac{\pi}{3}, \frac{2\pi}{3}$

6) $\cos 2x = \frac{\sqrt{3}}{2}$

A) $\frac{3\pi}{2}$
B) $\frac{\pi}{2}$
C) $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$
D) $\frac{\pi}{6}, \frac{11\pi}{6}$

Solve the equation on the interval [0, 2π).

7) $\tan x\cos x + \tan x + \cos x + 1 = 0$

A) $\frac{3\pi}{4}, \pi, \frac{7\pi}{4}$
B) $0, \frac{3\pi}{4}, \frac{7\pi}{4}$
C) $\frac{3\pi}{4}, \frac{7\pi}{4}, 2\pi$
D) $0, \frac{\pi}{4}, \frac{5\pi}{4}$

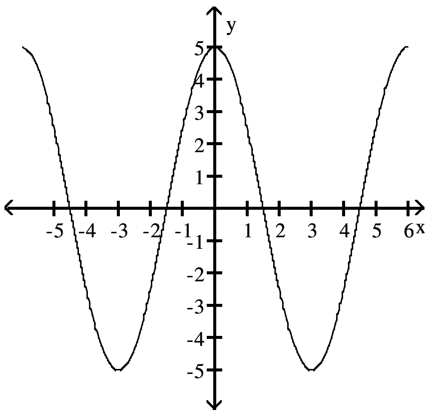
Use a right triangle to write the expression as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

8) $\sin(\tan^{-1} x)$

A) $\frac{\sqrt{x^2 + 1}}{x^2 + 1}$
B) $\frac{x\sqrt{x^2 + 1}}{x^2 + 1}$
C) $x\sqrt{x^2 + 1}$
D) $\frac{x\sqrt{x^2 - 1}}{x^2 - 1}$

Find an equation for the graph.

9)



A) $y = 3 \cos 5\pi x$ B) $y = 5 \cos 3\pi x$ C) $y = 3 \cos \frac{\pi}{5} x$ D) $y = 5 \cos \frac{\pi}{3} x$

Find the standard form of the equation of the ellipse satisfying the given conditions.

10) Major axis horizontal with length 20; length of minor axis = 12; center (0, 0)

A) $\frac{x^2}{100} + \frac{y^2}{36} = 1$ B) $\frac{x^2}{400} + \frac{y^2}{144} = 1$ C) $\frac{x^2}{20} + \frac{y^2}{36} = 1$ D) $\frac{x^2}{36} + \frac{y^2}{100} = 1$

Convert the equation to the standard form for a parabola by completing the square on x or y as appropriate.

11) $y^2 + 2y + 4x - 3 = 0$

A) $(y - 1)^2 = -4(x - 1)$ B) $(y + 1)^2 = -4(x + 1)$ C) $(y + 1)^2 = -4(x - 1)$ D) $(y - 1)^2 = 4(x - 1)$

Find the location of the center, vertices, and foci for the hyperbola described by the equation.

12) $\frac{(x + 4)^2}{4} - \frac{(y + 2)^2}{64} = 1$

A) Center: $(-4, -2)$; Vertices: $(-6, -2)$ and $(-2, -2)$; Foci: $(-4 - 2\sqrt{17}, -2)$ and $(-4 + 2\sqrt{17}, -2)$

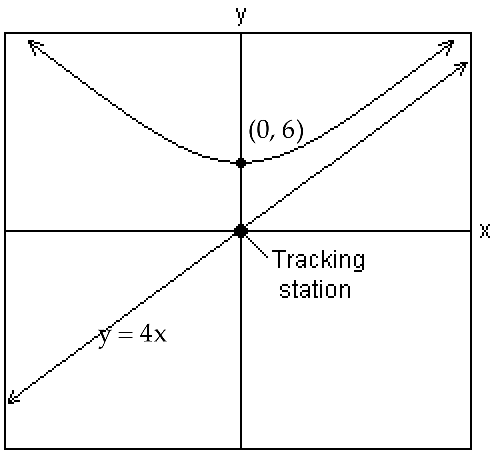
B) Center: $(4, 2)$; Vertices: $(2, 2)$ and $(6, 2)$; Foci: $(4 - 2\sqrt{17}, 2)$ and $(4 + 2\sqrt{17}, 2)$

C) Center: $(-4, -2)$; Vertices: $(-5, -2)$ and $(-1, -2)$; Foci: $(-3 + 2\sqrt{17}, -1)$ and $(-1 + 2\sqrt{17}, -1)$

D) Center: $(-4, -2)$; Vertices: $(-6, 2)$ and $(-2, 2)$; Foci: $(-4 - 2\sqrt{17}, 2)$ and $(-4 + 2\sqrt{17}, 2)$

Solve the problem.

13) A satellite following the hyperbolic path shown in the picture turns rapidly at (0, 6) and then moves closer and closer to the line $y = 4x$ as it gets farther from the tracking station at the origin. Find the equation that describes the path of the satellite if the center of the hyperbola is at (0, 0).



A) $\dfrac{x^2}{36} - \dfrac{y^2}{\left(\frac{24}{3}\right)^2} = 1$ B) $\dfrac{y^2}{\frac{9}{4}} - \dfrac{x^2}{36} = 1$ C) $\dfrac{y^2}{36} - \dfrac{x^2}{\frac{9}{4}} = 1$ D) $\dfrac{x^2}{\left(\frac{24}{3}\right)^2} - \dfrac{y^2}{36} = 1$

Rewrite the equation in a rotated x'y'-system without an x'y' term. Express the equation involving x' and y' in the standard form of a conic section.

14) $31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0$

A) $\dfrac{x'^2}{9} + \dfrac{y'^2}{4} = 1$ B) $y'^2 = -4\sqrt{2}x'$ C) $x'^2 = -4\sqrt{2}y'$ D) $\dfrac{x'^2}{4} + \dfrac{y'^2}{9} = 1$

Eliminate the parameter t. Find a rectangular equation for the plane curve defined by the parametric equations.

15) $x = 7 \cos t$, $y = 7 \sin t$; $0 \le t \le 2\pi$

A) $x^2 - y^2 = 49$; $-7 \le x \le 7$ B) $x^2 + y^2 = 7$; $-7 \le x \le 7$
C) $x^2 + y^2 = 49$; $-7 \le x \le 7$ D) $x^2 - y^2 = 7$; $-7 \le x \le 7$

Solve the problem.

16) A radio transmission tower is 250 feet tall. How long should a guy wire be if it is to be attached 12 feet from the top and is to make an angle of 30° with the ground?

A) 500.0 feet B) 476.0 feet C) 288.7 feet D) 274.8 feet

An object is attached to a coiled spring. The object is pulled down (negative direction from the rest position) and then released. Write an equation for the distance of the object from its rest position after t seconds.

17) amplitude = 9 cm; period = 3 seconds

A) $d = -9 \cos \dfrac{\pi}{3}t$ B) $d = -9 \cos \dfrac{2}{3}\pi t$ C) $d = -9 \sin \dfrac{2}{3}\pi t$ D) $d = -3 \cos \dfrac{2}{9}\pi t$

Find the exact value of the expression. Do not use a calculator.

18) $1 + \sin^2 75° + \sin^2 15°$

A) −1 B) 2 C) 1 D) 0

Find the domain of the composite function $f \circ g$.

19) $f(x) = \frac{5}{x + 7}$, $g(x) = \frac{7}{x}$

A) $(-\infty, 0) \cup (0, -1) \cup (-1, \infty)$
B) $(-\infty, \infty)$
C) $(-\infty, -7) \cup (-7, 0) \cup (0, \infty)$
D) $(-\infty, -7) \cup (-7, -1) \cup (-1, 0) \cup (0, \infty)$

Express the given function H as a composition of two functions f and g such that $H(x) = (f \circ g)(x)$.

20) $H(x) = \sqrt{5 - \sqrt{x - 5}}$

A) $f(x) = \sqrt{5 + x}$, $g(x) = \sqrt{x - 5}$
B) $f(x) = \sqrt{x - 5}$, $g(x) = \sqrt{5 - x}$
C) $f(x) = \sqrt{x - 5}$, $g(x) = \sqrt{x - 5}$
D) $f(x) = \sqrt{5 - x}$, $g(x) = \sqrt{x - 5}$

Find the vertical asymptote(s), if any, of the graph of the rational function.

21) $f(x) = \frac{x^2 + 7x}{x^2 - 2x - 63}$

A) $x = 9$
B) $x = -9$, $x = 7$
C) $x = 9$, $x = -7$
D) no vertical asymptote

Find the polynomial P(x) with real coefficients having the specific degree, leading coefficient, and zeros.

22) degree: 3, leading coefficient: –3, zeros: 3, 4 + 6i

A) $3x^3 + 33x^2 + 228x + 468$
B) $x^3 - 11x^2 + 76x - 156$
C) $-3x^3 + 33x^2 - 228x + 468$
D) $-3x^3 - 33x^2 - 228x - 468$

Determine whether the given function is one–to–one. If it is one–to–one, find its inverse.

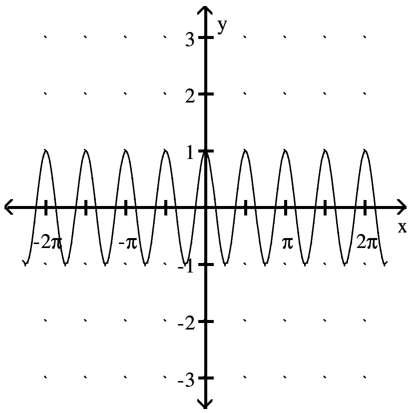
23) $f(x) = \frac{7}{x - 8}$

A) $f^{-1}(x) = \frac{8x + 7}{x}$
B) Not one–to–one
C) $f^{-1}(x) = \frac{-8 + 7x}{x}$
D) $f^{-1}(x) = \frac{x}{-8 + 7x}$

Match the function with its graph.

24) 1) $y = \sin(4x)$ 2) $y = 4\cos(x)$
3) $y = 4\sin(x)$ 4) $y = \cos(4x)$

A)



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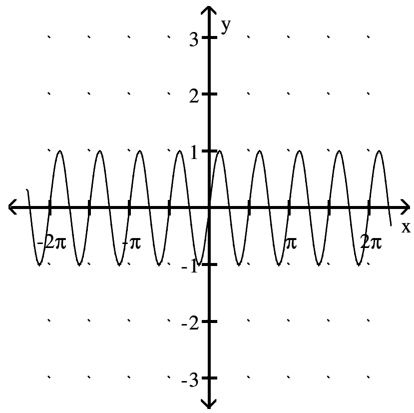
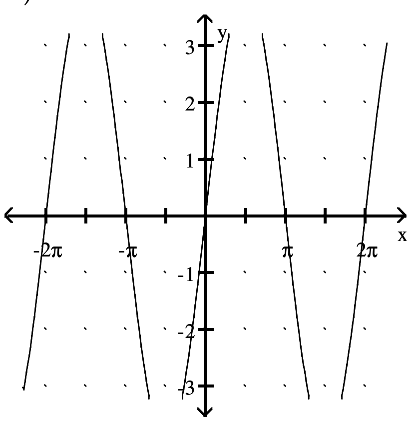
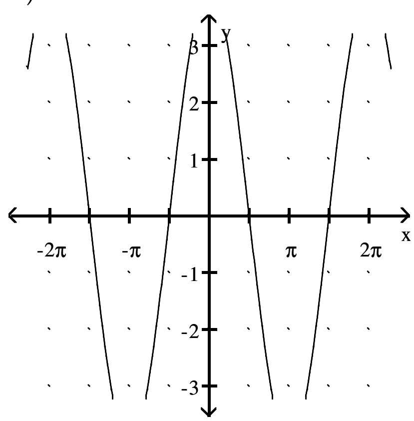


C)



D)



A) 1A, 2C, 3D, 4B B) 1A, 2D, 3C, 4B C) 1A, 2B, 3C, 4D D) 1B, 2D, 3C, 4A

Solve the equation.

25) $\frac{\pi}{3} + \arccos(2x) = \frac{\pi}{6}$

A) $x = \frac{\sqrt{3}}{2}$ B) $x = \frac{\sqrt{3}}{4}$ C) $x = -\frac{\sqrt{3}}{4}$ D) $x = \frac{1}{4}$

Solve the problem.

26) A domed ceiling is a parabolic surface. For the best lighting on the floor, a light source is to be placed at the focus of the surface. If 7 m down from the top of the dome the ceiling is 7 m wide, find the best location for the light source.

A) 0.4 m down from the top B) 1.2 m down from the top
C) 1.8 m down from the top D) 0.8 m down from the top

Identify the conic section represented by the equation.

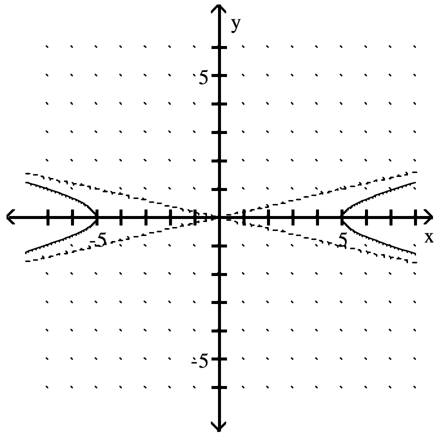
27) $x^2 + y^2 + 6x - 8y + 13 = 0$

A) parabola B) ellipse C) hyperbola D) circle

Graph the hyperbola.

28) $y^2 - \frac{x^2}{25} = 1$

A)



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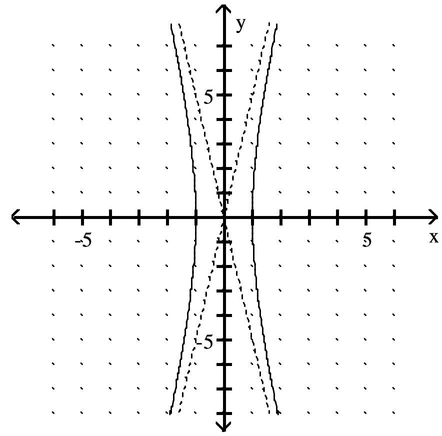

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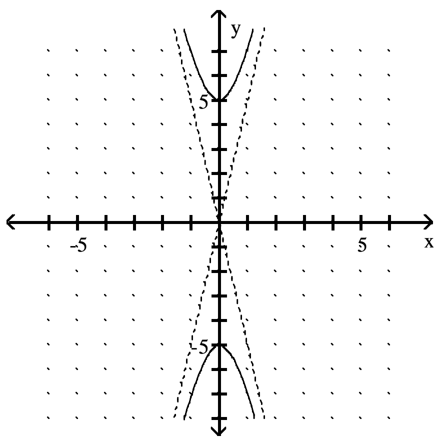
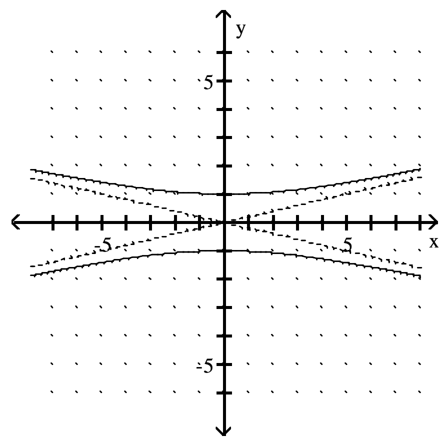


D)



Solve the logarithmic equation.

29) $\log_4 (x - 2) + \log_4 (x - 2) = 1$

A) 4 B) −4, 4 C) $-\sqrt{5}, \sqrt{5}$ D) $\sqrt{5}$

Expand the expression.

30) $\ln \sqrt[3]{\frac{(x^2 + 1)^9}{2x + 5}}$

A) $6 \ln x - \frac{1}{3} \ln 2x - \frac{1}{3} \ln 5$

B) $3 \ln (x^2 + 1) + \ln (2x + 5)$

C) $3 \ln (x^2 + 1) - \frac{1}{3} \ln (2x + 5)$

D) $\frac{3 \ln (x^2 + 1)}{\ln (2x + 5)}$

Find the domain of f and write it in interval notation.

31) $f(x) = 3 \log (x^2)$

A) $\left(\frac{27}{2}, \infty\right)$ B) $(-\infty, 0) \cup (0, \infty)$ C) $(0, \infty)$ D) $(3, \infty)$

Solve the problem.

32) Technology matrix A, representing interindustry demand, and matrix D, representing consumer demand for the sectors in a two–sector economy, are given. Use the matrix equation $X = (I - A)^{-1}D$ to find the production level X that will satisfy both demands.

$$A = \begin{bmatrix} 0.1 & 0.4 \\ 0.5 & 0.2 \end{bmatrix}, D = \begin{bmatrix} 45 \\ 27 \end{bmatrix}$$

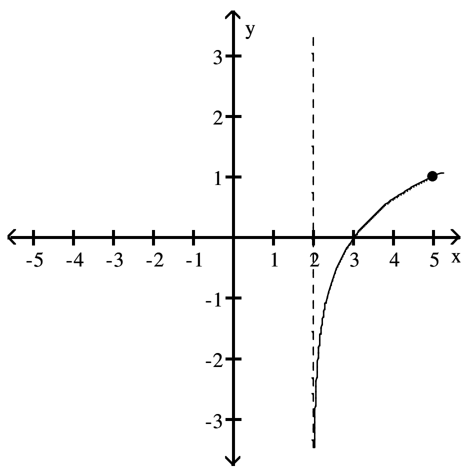
A) $X = \begin{bmatrix} 90 \\ 90 \end{bmatrix}$

B) $X = \begin{bmatrix} 100 \\ 90 \end{bmatrix}$

C) $X = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$

D) $X = \begin{bmatrix} 90 \\ 100 \end{bmatrix}$

33) The graph of a logarithmic function of the form $y = \log_a(x - c)$ is given. Use the graph to determine a and c.



A) a = 3; c = 1

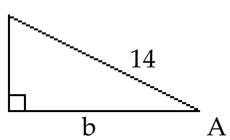
B) a = 3; c = 2

C) a = 4; c = 2

D) a = 3; c = 3

Use the inverse trig functions to express the angle in terms of the indicated unknown side.

34)



Use one of the inverse trig functions $\csc^{-1}$ or $\sec^{-1}$ to express angle A in terms of b.

A) $A = \csc^{-1} \frac{14}{b}$

B) $A = \sec^{-1} \frac{b}{14}$

C) $A = \sec^{-1} \frac{14}{b}$

D) $A = \csc^{-1} \frac{b}{14}$

Which answer choice is equivalent to the given expression?

35) $\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}$

A) $\sec^2 x - \sin x$

B) $2 \csc^2 x$

C) $4 \tan^2 x$

D) $2 \sec^2 x$

Answer Key
Testname: PRECALCULUS INDIVIDUAL EXAM - MATH COMPETITION 2008

1) A
2) A
3) B
4) D
5) C
6) C
7) A
8) B
9) D
10) A
11) C
12) A
13) C
14) D
15) C
16) B
17) B
18) B
19) A
20) D
21) A
22) C
23) A
24) D
25) B
26) A
27) D
28) D
29) A
30) C
31) B
32) A
33) B
34) C
35) D