This is an on-line appendix for use with
Frequency Synthesis by Phase Lock, 2nd Ed.[1]

Appendix 8.D Offset Reference and Diophantine Synthesis

In Section 8.2.3, we discuss an architecture called Offset Reference, which is intended to produce finer frequency steps with wider loop bandwidth. It combines the outputs from two loops but has an advantage over other dual loops, in which one loop output is frequency divided before being mixed with the other, because the output frequencies of these individual loops tend to be more commensurate and, therefore, more suitable for mixing. We found that developing a tuning algorithm for the Offset Reference loop can be require some effort. A synthesis method [Sotiriadis, 2006] called Diophantine, after the mathematics that has been applied to it, is essentially the same as Offset Reference but is more restrictive. The restrictions facilitate the specification of design parameters and of a tuning algorithm. In addition, it has been applied to synthesizer architectures for more than two loops.

8.D.1 Two-Loop Synthesizers

Sotiriadis has shown that divide ratios N_1 and N_2 can be found that generate any frequency multiple of Δf (Fig. 8.12), which should come as no surprised from the description of the method and the example in Section 8.2.3. He has also found that the frequency range

$$-F_{\text{ref},0} \le \left(f_{\text{out}} - \bar{f}_{\text{out}}\right) \le F_{\text{ref},0}\,. \tag{8.D.1}$$

can be synthesized using individual-loop output frequencies f_i with a range given by

$$-F_{\text{ref},0} \le \left(f_i - \bar{f}_i\right) \le F_{\text{ref},0}\,,\; i = 1,2, \tag{8.D.2}$$

which can also be stated

$$-R_i \le \left(N_i - \bar{N}_i\right) \le R_i\,. \tag{8.D.3}$$

Here R_i is the prescaler ratio for loop i and all pairs of R_i are relatively prime (their greatest common divisor is 1; no number but 1 can divide into both without a remainder). While there is only one pair under consideration now, the same requirement holds for multiple loops to be discussed below.

The over bars above indicate the mean of the ranges. For Fig. 8.12 and Table 8.4, Eq. (8.D.1) implies a total range of

$$2F_{\text{ref},0} = 2(255)(256)\,\Delta f = 130{,}560\,\Delta f \tag{8.D.4}$$

for the output and for each of the oscillators. As can be seen from Table 8.4, the actual ranges are slightly smaller since the range of N was limited to 500 for each oscillator whereas Eq. (8.D.3) calls for

$$\Delta R_i \leq 2(R_i) + 1, \tag{8.D.5}$$

which can be 511 and 513 respectfully for the two loops. The range of f_{out} is also slightly smaller than the ranges of f_i in Table 8.4.

An advantage of designing for the maximum range of Eq. (8.D.1) is that a tuning algorithm exists, as given in the MATLAB script loop2tune.m .*

8.D.2 Multi-Loop Synthesizers

More than two loops can be combined. The reference frequency is

$$F_{ref,0} = R_1 R_2 ... R_k \, \Delta f_{out} \, . \tag{8.D.6}$$

If the output frequency range is restricted to that given by Eq. (8.D.1), the individual oscillator frequencies will have ranges given by Eq. (8.D.2), which is equivalent to Eq. (8.D.3), and the tuning algorithm is given by the MATLAB script loopxtune.m.* Multiple loops can achieve very fine tuning without sacrificing loop bandwidth.

The frequencies can be combined, mixing two signals at a time [Sotiriadis, 2008], the IF produced by mixing one pair then mixed with the next frequency. It is important to choose the mean frequencies to avoid spurious mixer responses and mixing only two signals at a time aids this by allowing filtering after each mixer.

8.D.3 Signal Mixing

Let us look at the mixing process for Fig. 8.12 and Table 8.4, which illustrates some of the consideration for both double and multi-loop synthesizers. Table 8.1 was used for preliminary considerations in choosing the architecture of Fig. 8.12. Of greatest concern there was a -2 by 3 (LO multiple by RF multiple) crossover spur.

Fig. 8.D.1 shows a normalized spur plot [Egan, 2003] of the type shown in Fig. 1.8. The synthesized frequency is the LO, f_1 is the RF input to the mixer and the IF is f_2 in Fig. 8.12. Table 8.D.1 summarizes the frequencies from Tables 8.3 and 8.4.

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* * Click on hyperlink in *Frequency Synthesis by Phase Lock* page.

	Loop 1	Loop 2	Output
	IF	RF	LO
minimum	256.0	1275.0	1596.0
maximum	384.0	1402.5	1721.7
mean	320.0	1338.8	1658.9
range	128.0	127.5	125.7

Table 8.D.1 Frequencies of Synthesizer in Fig. 8.12 in units of 1000Δ, where Δ is the synthesizer resolution (step size).

Rectangle 1 represents the frequency ranges of the mixer inputs at the maximum IF frequency, rectangle 2 is at the minimum IF, and rectangle 3 is at the mean IF. While the rectangles represent the total ranges of the frequencies, only frequency pairs that lie on the green 1x1 line actually occur. If the frequency pairs represented by Tables 8.3 and 8.4 were plotted, all of the points would lie on the 1x1 curve. Rectangle 1 encloses the region of the 1x1 curve that applies whenever f_1 is at its maximum value, something that occurs repeatedly in the tables. As the synthesizer progresses through its frequencies in order, the vertical displacement in each rectangle progresses smoothly while the applicable rectangle changes repeatedly and the RF frequency is determined by the intersection of the LO frequency with the 1x1 curve in the applicable rectangle. The closest that the operating point can come to the -2x3 crossover spur at (3,4) is at the intersection of the 1x1 curve and rectangle 1 and there may not be an operating point just there. Nevertheless, we expect one to occur close to that point and it is easier to work with that intersection than to search for the nearest operating point. The intersection has f_{LO} = 1596 and f_{RF} = 1275 in units of 1000Δ . The -2x3 spur will produce an IF of ($2f_{LO} - 3f_{RF}$) = 633 at the same frequencies. The guard band width is (633-389) = 344 and the required IF filter half bandwidth is (398-322.5) = 75.5 so the required IF filter shape factor is a rather generous 4.6 (= 344/75.5).

The spur chart of Fig. 8.D.1 represents mixing within the loop, as shown in Figure 8.12 . The output frequency is still the sum of the two frequencies, f_1 and f_2, that are input to the dividers $\div N_1$ and $\div N_2$, as if the frequencies had each been generated by a VCO and mixed outside of the loop, as in Fig. 8.11. Figure 8.D.2 shows the spur chart for that configuration. Table 8.D.1 applies again except that the first column is RF, the second is LO, and the third is IF. We see that there is a crossover spur within the range of frequencies used in synthesis. This is the same order and estimated level as in the first case but it cannot be filtered out. If this level is too large, the level of the RF input would have to be dropped; the spur would fall at thrice the rate of the desired output, causing their ratio to change 2 dB per dB of power reduction.

We might change the median frequencies of the two loops in this second case, although both conversions are within generally advantageous regions of the spur chart (Regions 2 and 3 respectively in Fig. 7.29 of reference [4]). Again in Fig. 8.D.2, only half of the LO and RF ranges are used at the extreme IFs but the whole range is used at the mean IF. In this case the output is the IF so, as the output progresses uniformly, the rectangles will step from #4 to #5 in Fig. 8.D.2 while the operating point moves about on the 1x1 curve within each current rectangle.

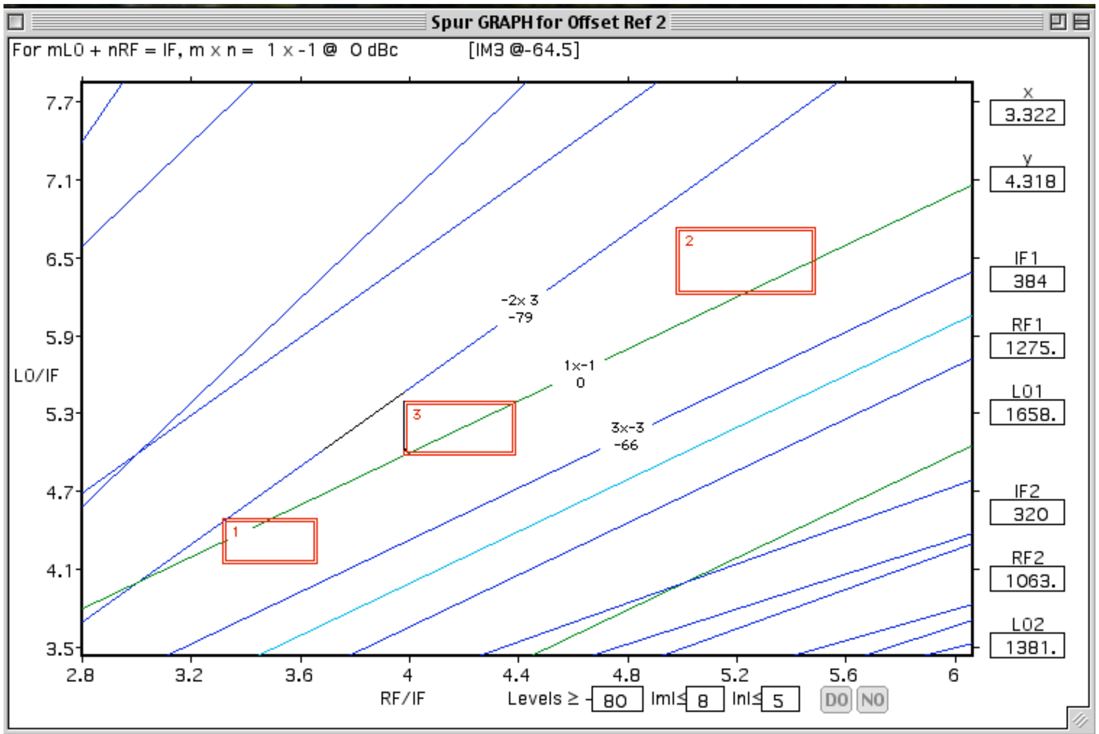

Fig. 8.D.1 Spur chart for Mixing Inside Loop. Cursor is at intersection of 1x-1 green line and left side of rectangle 1 but does not appear in this view. Mixer parameters are extrapolated for an M1 mixer at 12 dBm LO and -16 dBm RF input.

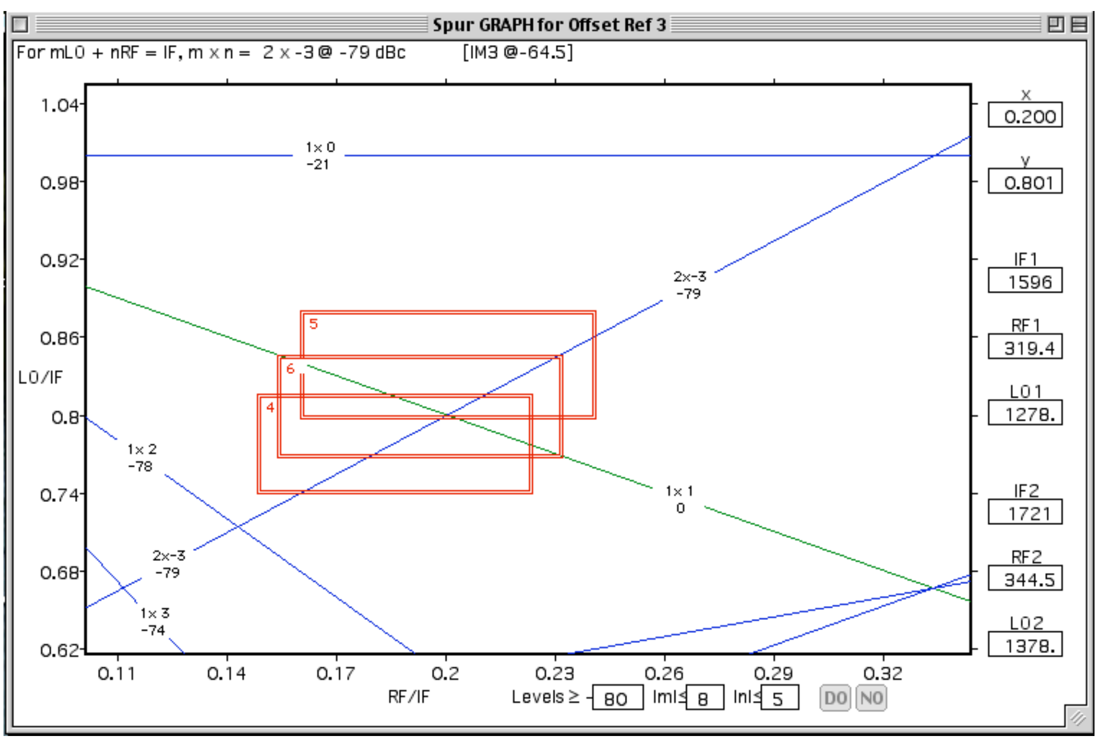

Fig. 8.D.2 Spur Chart for Mixing Outside of Loops. Cursor is at intersection of 1x1 and 2x-3 crossover but does not appear in this view. M1 mixer at 12 dBm LO with -16 dBm RF input.

[1] Egan, W. (2000) *Frequency Synthesis by Phase Lock, 2nd Ed.* New York: Wiley.

[2]Sotiriadis, P. (2006) "Diophantine Frequency Synthesis," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control,* vol. 53, no. 11, November, pp. 1988-1998.

[3] Sotiriadis, P. (2008) "Cascaded Diophantine Frequency Synthesis," *IEEE Transactions on Circuits and Systems—I: Regular Papers,* vol. 55, no. 3, April, pp. 741-751.

[4] Egan, W. (2000) *Practical RF System Design*. New York: Wiley, Ch. 7.