This is an on-line appendix for use with
Practical RF System Design

Appendix D: Double-Sideband Noise Figure

"Double-sideband" (DSB) noise figure (DSBNF), or factor (DSBNf), can be a useful concept. When this term is used, the noise figure F, as we have discussed it, may then be referred to as the single-sideband (SSB) noise figure (SSBNF).

DSBNF arises in cases where frequencies at both

$$f_{R+} = \mathrm{f}_L + f_I \quad \text{(D.1)}$$

and

$$f_{R-} = f_L - f_I \quad \text{(D.2)}$$

are converted to f_I. The name may arise from the fact that f_{R+} and f_{R-} could be sidebands on a carrier whose frequency equals f_L. If an IF band is defined by a range of f_I, the corresponding RF signal and image bands are defined by Eqs. (D.1) and (D.2), not necessarily respectively.

For both types of noise figure,

$$f = \frac{N_{\text{out}}}{N_T}\frac{S_{\text{in}}}{S_{\text{out}}}, \quad \text{(D.3)}$$

where N, and possibly S, is an average density over the same bandwidth at input and output. SSBNf is taken with S_{in} present in only the signal input band, but the DSBNf is taken with S_{in} present in both the signal and image bands. That tends to make S_{out} about twice as large for DSBNf, leading to a 3-dB reduction in DSBNF relative to SSBNF. In other words, the DSBNF is lower because S_{in} is only half of the input signal power.

There are cases where the lower DSBNF is a better representation of a system's characteristic than the SSBNF and there are cases where it is not.

If the desired signals consist of broadband noise, it may be beneficial to permit the noise to enter the system at both sideband frequencies. Two systems in which this occurs are the radiometer, whose purpose is to measure noise power density, and one common noise figure test setup for a mixer. We will consider these two systems, then compute the relationship between DSBNF and SSBNF in the general case, and finally consider other possible uses for DSBNF.

D.1 Radiometer

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A radiometer can be constructed so it is represented by Fig. 3.11, for which we can compute f_{cas} from Eq. (3.44). Assume, for the moment, that the filter is ideal, having no loss at the signal frequency and infinite attenuation at the image frequency. Assume that other gains and noise factors are the same at signal and image frequencies. The best noise factor f_{cas} will occur if the mixer is preceded directly by the filter, so no image noise (beyond N_T) enters the mixer, and is given by Eq. (3.51). If, on the other hand, we move the filter to the front of the chain, the image noise that is added to the chain at the mixer will be as great as the noise that accompanies the signal. If the gain preceding the mixer is high (so the noise figure depends mainly on that part of the system), this will nearly double the value of f_{cas}. If we now eliminate the filter, f_{cas} will not change because there will still be thermal noise at the input to the image chain and because S_{in}, in Eq. (D.3), does not change. However, if S_{in} exists in the image band, removing the filter will double the true input power and the output *S/N*, making it at least as good as when the filter just preceded the mixer.

In fact, with finite gains preceding the mixer, the noise figure of the back end of the chain will also contribute to the system noise figure, so the image noise would have degraded the output S/N by less than 3 dB. Nevertheless, doubling the input signal power will improve the output S/N by 3 dB. Thus, assuming equal responses at signal and image, the output S/N would be better when the image is not filtered out than when it is, even if the filter would be just before the mixer. Moreover, eliminating the filter also eliminates the impossible task of finding the ideal filter that we had assumed, and so, in practice, would further improve the *S/N*.

Figure D.1 shows two systems, one that passes only one sideband and the second that passes both. Using some ideal components, assuming for the mixer a DSBNF of 0 dB and a SSBNF of 3 dB, system noise figures are shown in Fig. D.2.

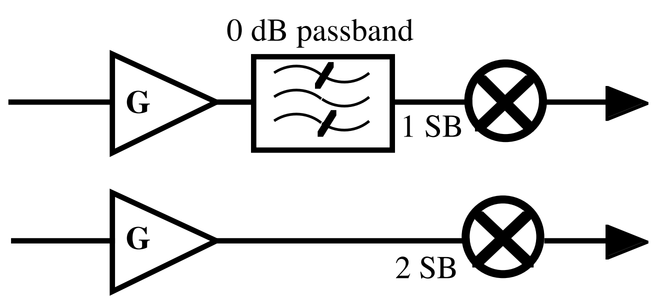

Fig. D.1 Two systems.

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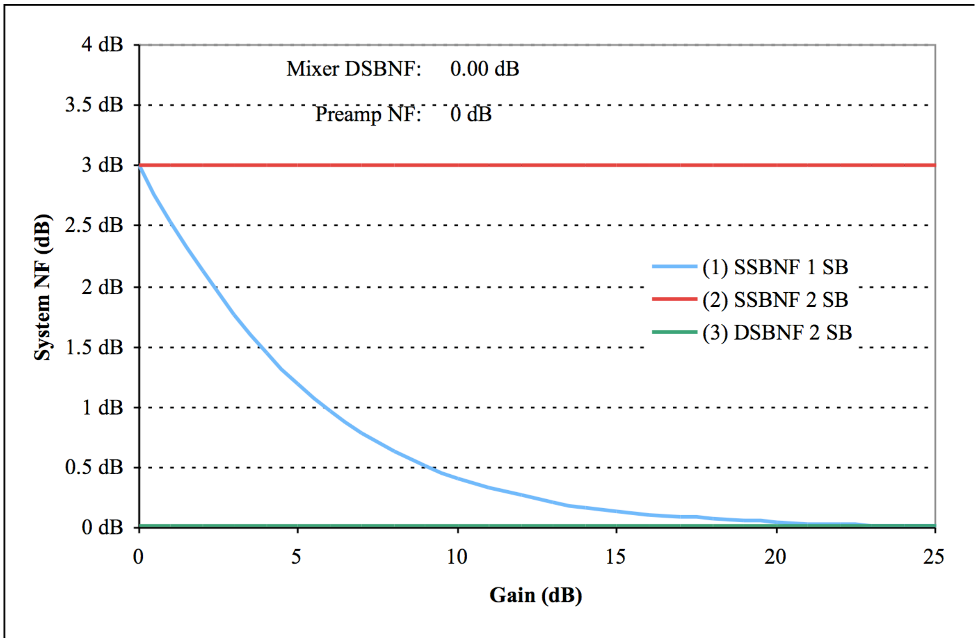

Fig. D.2 System Noise Figures

We see that, with unity amplifier gain, the SSBNF is 3 dB for both systems, but, for the 1-sideband system (1), it drops toward 0 dB at higher gains. This ideal mixer has $F = -3$ dB, because only half of the signal power is converted to the IF[1], making it a 3-dB attenuator. That noise factor is divided by the amplifier gain in the 1-sideband system (1) (Eq. 3.44), but, in the 2-sideband system, the image noise is also amplified by that gain, keeping the SSBNF constant at 3 dB (2) (Eq. 3.48). (Curve 2 also represents the SSB system with the ideal filter preceding the gain.) However, the output S/N of the 2-sideband system is better indicated by the DSBNF (3) because it represents the total output signal power (rather than that due to just the input at either the signal or image).

In determining f_{cas}, we inject a single signal into the system's front end and measure signal and noise powers at its output. When we remove the filter in the operational environment of the radiometer, we double the relevant output signal power because the signal of interest is the input noise power density, which exists at both the signal and image frequencies. The proper measure of the quality of this system is the ratio of the signal power density to noise power density at the output divided by the ratio of signal power density, which the system is designed to measure, to thermal noise power density at the input. To a first approximation, we get the same output S/N, whether we use an optimum SSB system (curve 1 at high gains) or a DSB system, but the DSB system can have a much higher f (curve 2). Therefore the DSBNF is a better indication of the quality of this DSB system than is the SSBNF. Nevertheless, the SSBNf is the f that is used in our equations in Chapter 3 and the DSBNf is not useful there. It should be applied to a *system* where appropriate as an alternative to just recognizing that the desired system output is better than would be indicated by f.

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DSBNF best represents the performance of the DSB system (2SB in Fig. D.1) and the SSBNF best represents the performance of the SSB system (1SB in Fig. D.1). The SSBNF would be 3 dB too high for this DSB system. The DSBNF doesn't apply to the 1SB system but, if the 2SB system were processing a signal in the signal band rather than noise in both bands, the DSBNF would be 3 dB too low to represent that system.

Figure D.3 is similar to Fig. D.2 but with more realistic component parameters. No noise has been added after the mixer but the effect of mixer noise figure is similar[2].

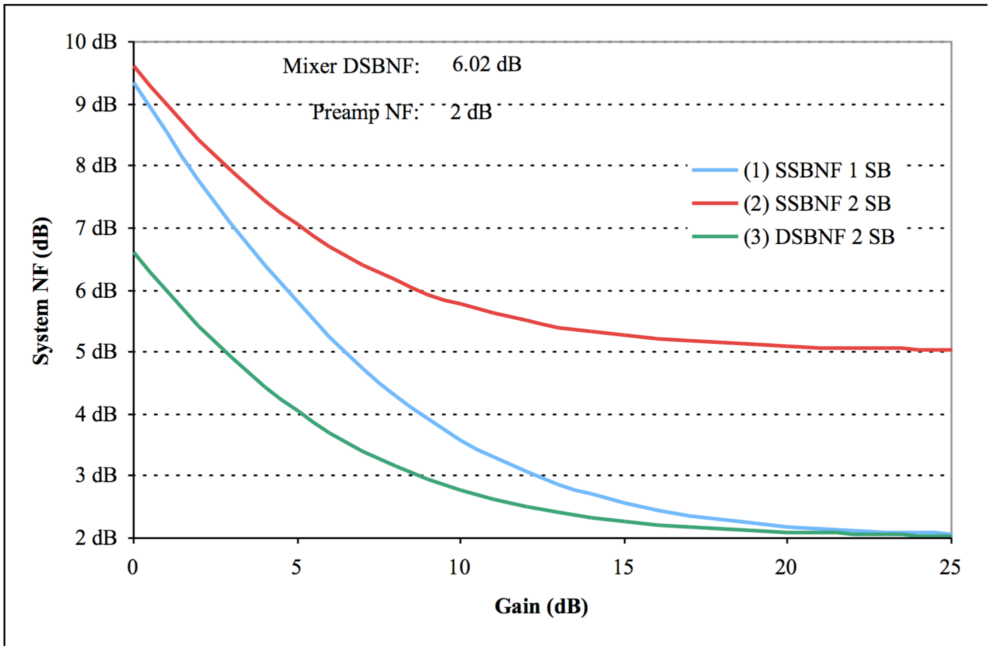

Fig. D.3 Figure D2 with non-ideal components.

D.2 Mixer Noise-Figure Measure

A common and convenient method for measuring noise factor f employs a hot noise source and a cold noise source. The hot noise source supplies noise power with a density N_H equal to thermal noise at its nominal temperature T_H while the cold source supplies N_C equal to thermal noise density at temperature T_C. For simplicity, assume that T_C equals room temperature, 290°K. The amount by which N_H exceeds N_C can then be considered the signal so

$$\left.\frac{S}{N}\right|_{\text{in}} = \frac{N_H - N_C}{N_C} = \frac{N_H}{N_C} - 1 = \frac{T_H}{T_C} - 1 = \frac{T_H}{290°\text{K}} - 1 \triangleq \text{ENR}_{\text{in}}, \tag{D.4}$$

where ENR is excess noise ratio. The measured output S/N would be

$$\left.\frac{\mathrm{S}}{\mathrm{N}}\right|_{\mathrm{out}} = \frac{N_{H\,\mathrm{out}} - N_{C\,\mathrm{out}}}{N_{C\,\mathrm{out}}} = \frac{N_{H\,\mathrm{out}}}{N_{C\,\mathrm{out}}} - 1 \triangleq \mathrm{ENR}_{\mathrm{out}} \tag{D.5}$$

and the noise factor is then

$$f = \mathrm{ENR}_{\mathrm{in}} / \mathrm{ENR}_{\mathrm{out}} \,, \tag{D.6}$$

but, not for a DSB system. For a DSB system, this method measures the DSBNF.

Note that, for the radiometer, the DSBNF is a valuable measure, whereas, for the mixer test, it is acceptable because of convenience.

D.3 Relationship between DSBNF and SSBNF.

Noise factor (SSBNf) is

$$f = \frac{S_{\mathrm{in}}}{N_T} \Big/ \frac{S_{\mathrm{out}}}{N_{\mathrm{out}}} = \frac{N_{\mathrm{out}}}{N_T} \Big/ \frac{S_{\mathrm{out}}}{S_{\mathrm{in}}} = \frac{N_{\mathrm{out}}}{g N_T} \tag{D.7}$$

where g is the signal power gain. DSBNf is

$$f_{\mathrm{DSB}} = \frac{N_{\mathrm{out}}}{g_1 N_T + g_2 N_T} \,, \tag{D.8}$$

so

$$\frac{1}{f_{\mathrm{DSB}}} = \frac{(g_1 + g_2) N_T}{N_{\mathrm{out}}} = \frac{1}{f_1} + \frac{1}{f_2} \,. \tag{D.9}$$

If $f_1 = f_2 = f$,

$$\frac{1}{f_{\mathrm{DSB}}} = \frac{2}{f} \tag{D.10}$$

or

$$f = f_{\mathrm{SSB}} = 2 f_{\mathrm{DSB}} \tag{D.11}$$

and the value of f for a mixer would be obtained by doubling the measured f_{DSB}.

D.4 Other Uses for DSBNF

In the two cases that we have discussed, noise power from two different sources is added in a mixer. The sources are uncorrelated because they are random noise in two different frequency bands. Noise powers can be added in another manner, for example in a power summer. The results will be similar as long as the noise sources are uncorrelated. If not,

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we will have to consider how the components at various frequencies interfere with each other. For example, if the sources are two antennas whose individual patterns overlap significantly, the result will be a new antenna pattern rather than just the summing of two noise powers.

Suppose we receive a CW signal (or an RF pulse or symbol or chip) by setting the LO frequency close to the signal frequency. If we have quadrature mixers, we can observe the IF rotating in one direction or the other, depending on the sign of the difference frequency. The ratio of the average signal power out of either mixer to the noise will be given by the input S/N (power of the CW signal divided by thermal noise) divided by f. The average can be taken over time or over phase. Noise factor f will be higher because the noise from both sidebands is added. Even if the difference frequency becomes zero, this will be true. However, the average signal power will then be over all possible phase differences rather than over time. When the phase difference becomes constant, the average will become unimportant; the actual phase will determine the output signal power. If the phase difference should be zero, the signal power would double relative to its average power and DSBNF might become a better indicator of output S/N (for the CW or during a pulse or symbol or chip). This is just because it is 3 dB higher than the SSBNF and that equals the processing gain that occurs when peak, rather than average, power pertains. Nevertheless, we would have to analyze the system using f to find either system SSBNF (i.e., f) or DSBNF and we would have to consider the signal processing to determine which is a better representation.

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[1] If $f_{\text{IF}} = |f_L \pm f_{\text{RF}}|$, half the power is converted to $|f_L \mp f_{\text{RF}}|$.

[2] Eq (3.44) is $f_{\text{cas}} = \ldots + \frac{1}{g_{B1} g_2 g_{B3}} \left[(f_{B4} - 1) + (f_{B5} - 1)/g_{B4} \right]$, where module *B4* is the mixer and *B5* is all following circuitry. If the mixer has a 9 dB loss (SSB) and noise figure, this is $f_{\text{cas}} = \ldots + \frac{1}{g_{B1} g_2 g_{B3}} \left[(8 - 1) + (f_{B5} - 1) 8 \right]$ so a contribution from the mixer (B4) is equal to one from the following circuitry if f_{B5} = 1.875 (F_{B5} = 2.7 dB).

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